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DEVELOPMENT OF THE CAPABILITY FOR SURFACE-WIND ANALYSIS ON A PORTABLE GRID (FIB/UV-PORT)

by

Bruce R. Mendenhall

Prepared for:

The Commanding Officer
Naval Environmental Prediction Research Facility
Monterey, California 93940

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DEVELOPMENT OF THE CAPABILITY FOR SURFACE-WIND ANALYSIS ON A PORTABLE GRID (FIB/UV-PORT)

1. Introduction

The basic analysis capability is the Fields-by-Information Blending (FIB) methodology developed by Meteorology International Incorporated. The FIB technique has been used successfully in many atmospheric and oceanic applications to date. These applications include the analysis of sea-surface temperature, sea-level pressure, surface winds, and ocean thermal-structure parameters. It has also been used in the diagnosis of clear-column radiances estimates from side-to-side scans of observed satellite radiances, for blending clear-column satellite radiance information with the more conventional atmospheric mass-structure model parameters and for oceanic depth-temperature profile retrievals from a specified mixture of upper-ocean thermal structure parameters and deep ocean climatology. In addition, a general scalar FIB capability has been formulated for analysis of weather elements (e.g., fog, cloudiness) and other scalar fields. These capabilities are documented in the list of references at the end of this report.

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The general applicability of the FIB methodology results from the inherent properties of the scheme--all information elements relevant to the spatial distribution of an object parameter may be brought to bear upon the resultant analysis in proportion to their information content relative to the resolution and specific objective of the analysis. Hence, object parameter estimates and structure or pattern information, estimates of parameters related to the object parameter via linear relationships and linear constraints on the resultant object parameter analysis may be

accommodated by the scheme. These estimates may come from current and near-current reports, current related analyses, and earlier analyses, climatology or forecasts.

In addition, the FIB methodology is conceptually open-ended in its ability to accept new information elements; and it is general in its application to grids of varying dimensions and resolution down to that minimum mesh length afforded by the input object parameter density and distribution.

2. Formulations

The surface wind analysis capability, FIB/UV, has been designed to use a concurrent sea-level-pressure analysis via a diagnostic balance relationship for first-guess information for both the winds and the spreading of the winds. The sea-level-pressure analysis capability FIB/P has been designed to provide this first-guess information for FIB/UV in that both analyses are done on a staggered grid (see Fig. 1), where the grid locations for u, v and p have been chosen for optimum use of the diagnostic relationship. In addition, use of this grid permits ordering of the unknown resultant object parameters so that the line successive-over-relaxation (SOR) technique may be used for solving the blending equations in both FIB/UV and FIB/P.

A detailed description of the solution of the blending system of equations by line SOR is included as Appendix A. Programming details are given in Appendix B. Although expressed with respect to blending for P^*, these details apply equally to blending for U^*, V^*.

Consequently, the FIB/UV and FIB/P analyses may be run in tandem--each FIB/UV analysis is preceded by a FIB/P analysis. This increases the information yield in FIB/UV in several respects. Firstly, a good first guess for the spreading parameters in FIB/UV is produced by using a concurrent pressure analysis of equivalent resolution. Secondly, it permits the assimilation of information available in the pressure portion of the reports. This information would be difficult to incorporate directly into the wind analysis. Thirdly, kinematical extrapolation in time, of sea-level-pressure fields for first-guess information to the sea-level-pressure analysis, is more valid than kinematical extrapolation of surface wind fields for first guess to the surface wind analysis.

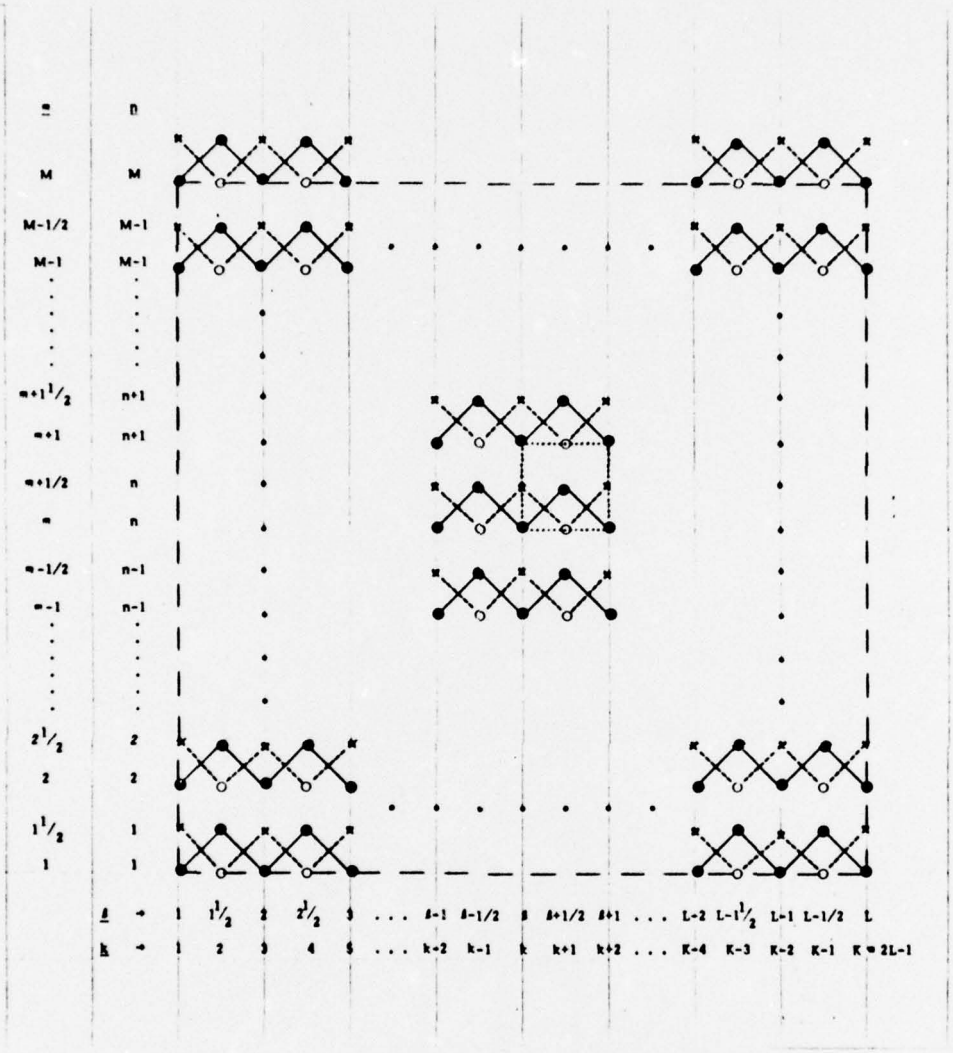

Fig. 1 The staggered grid showing reference locations for the pressure (•), the u component of wind (×) and the v component of wind (○). The ordering of the grid points in FIB/P is shown by the solid sawtooth line and in FIB/UV by the dashed sawtooth line. An arbitrary reference module is outlined with dotted lines. When referring to locations on the grid, the (ℓ, m) grid notation is used. When referring to the ordering of the points for blending, the (k,n) notation is used.

Schematics depicting the flow of operations in FIB/P and FIB/UV are shown in Figures 2 and 3, respectively. The information elements for FIB/P and FIB/UV are shown in Figures 4 through 7. Details of the FIB/P and FIB/UV analysis capabilities are given in references [31] and [27]. The present discussion will be limited to those adaptations necessary for application on the portable grid.

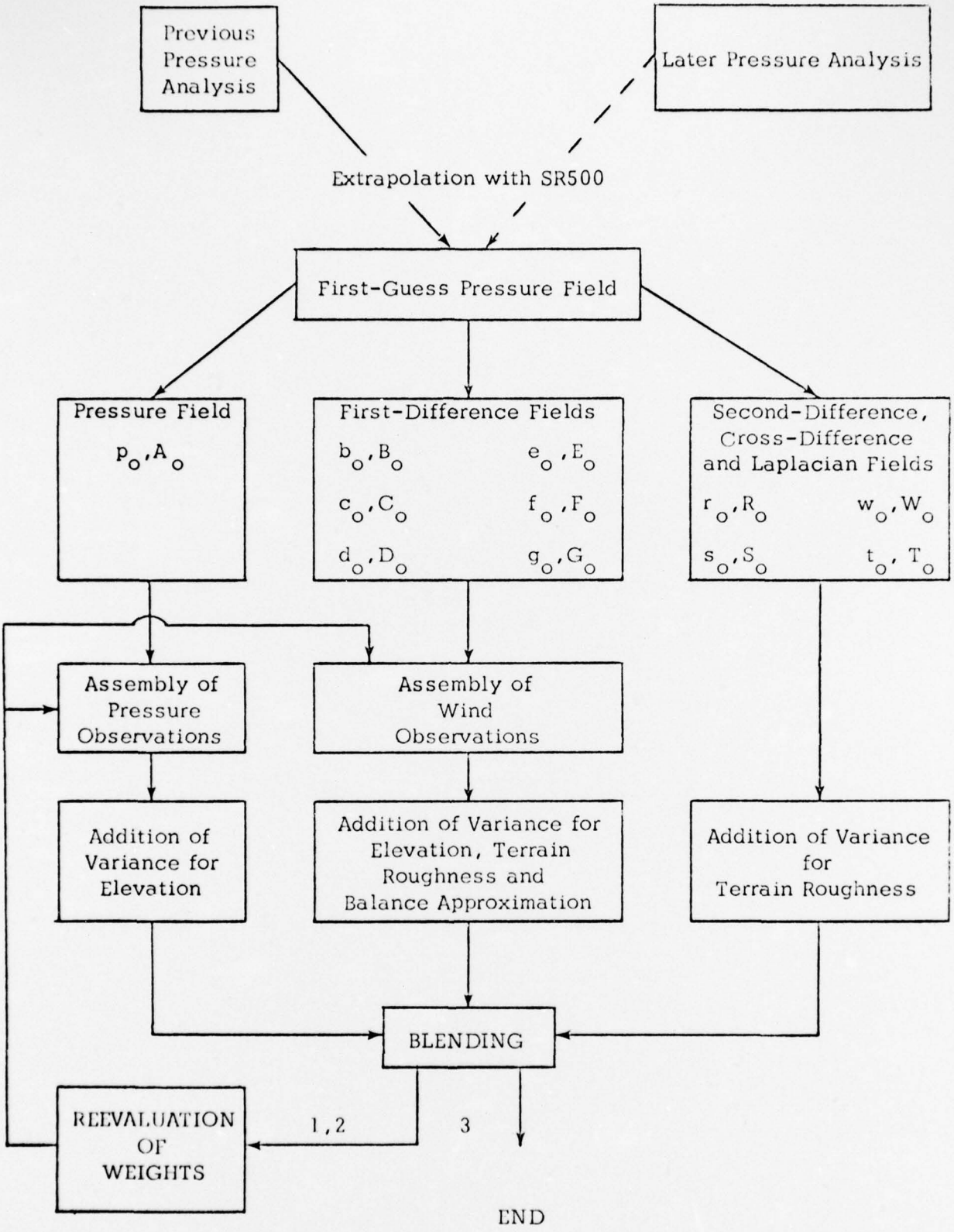

Fig. 2 Schematic Flow of Operations for FIB/P

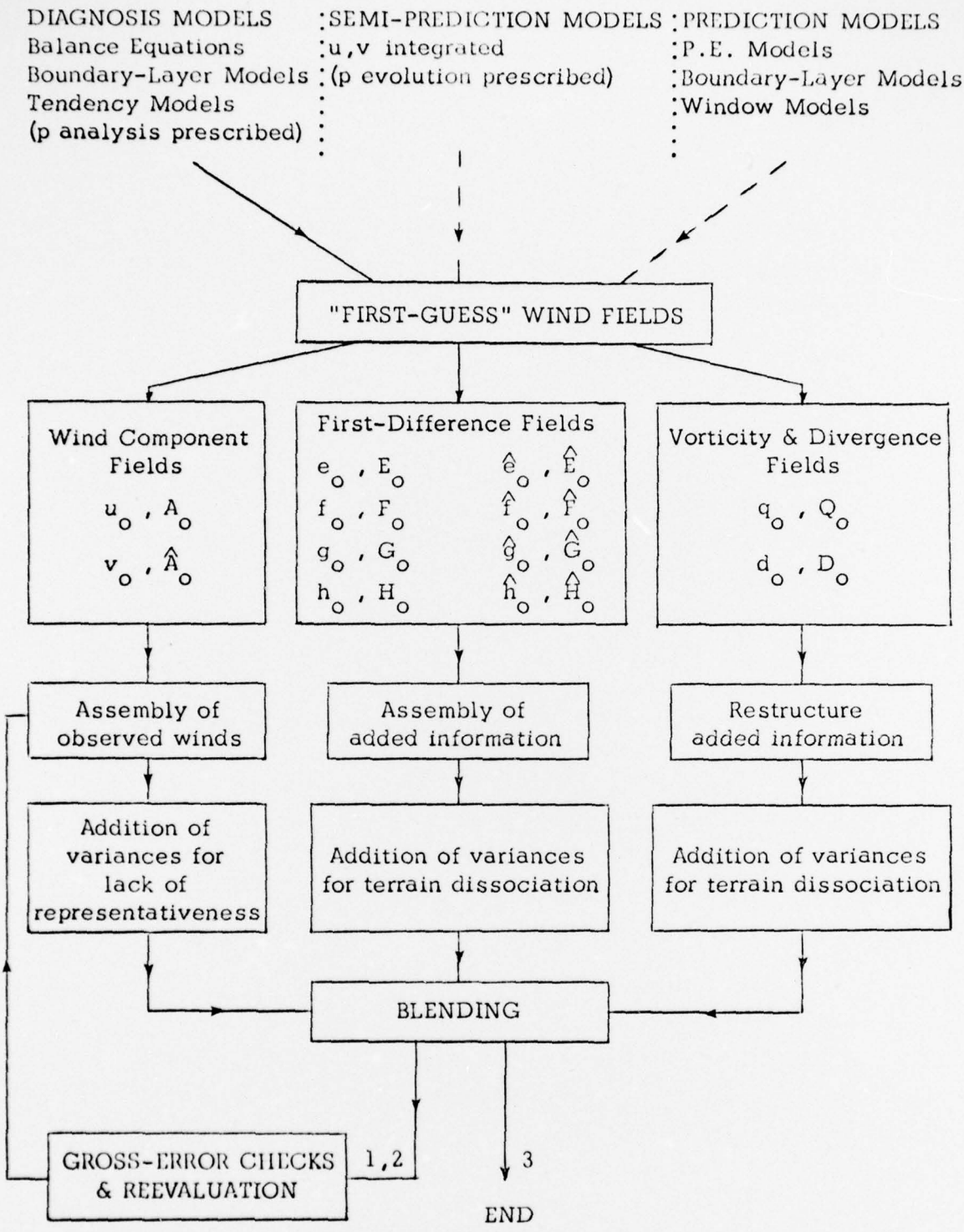

Fig. 3 Schematic Flow of Operations for FIB/UV

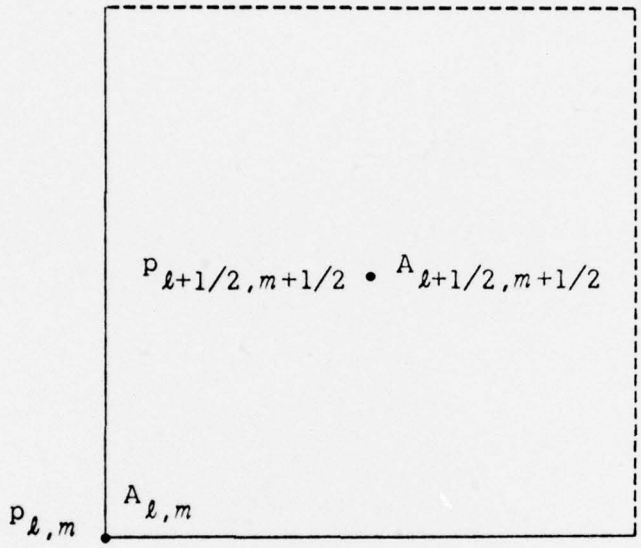

Fig. 4 The arbitrary FIB/P ℓ,m module for the staggered grid consisting of one corner point (ℓ,m), one center point $(\ell+1/2,m+1/2)$, two sides and the interior area. Reference points within the module are shown for the pressure, p, and associated weight, A.

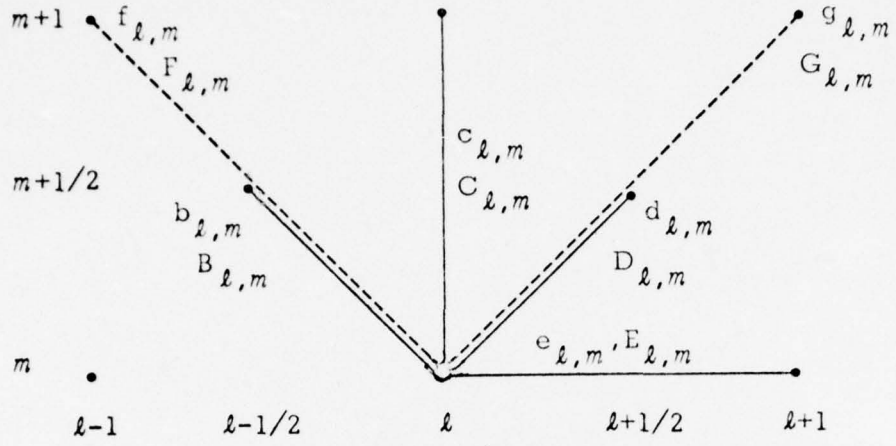

Fig. 5 Reference locations shown for the FIB/P first-difference spreading parameters defined below. The associated weights are given by the adjacent upper case letters. The indices ℓ and m assume both integer and half-integer values.

$$b_{\ell,m} \Rightarrow p_{\ell-1/2,m+1/2} - p_{\ell,m}$$

$$c_{\ell,m} \Rightarrow p_{\ell,m+1} - p_{\ell,m}$$

$$d_{\ell,m} \Rightarrow p_{\ell+1/2,m+1/2} - p_{\ell,m}$$

$$e_{\ell,m} \Rightarrow p_{\ell+1,m} - p_{\ell,m}$$

$$f_{\ell,m} \Rightarrow p_{\ell-1,m+1} - p_{\ell,m}$$

$$g_{\ell,m} \Rightarrow p_{\ell+1,m+1} - p_{\ell,m}$$

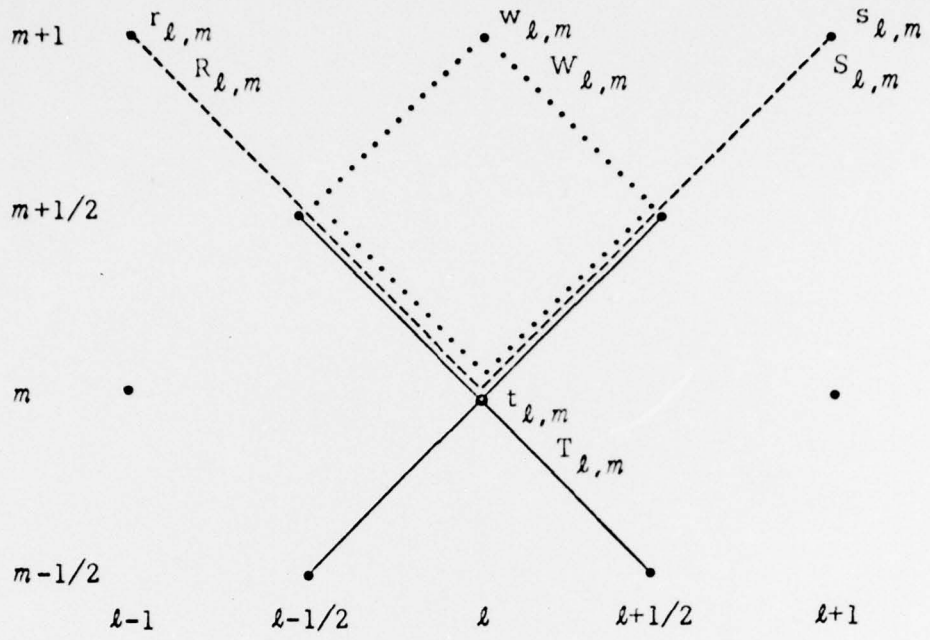

Fig. 6 Reference locations shown for the FIB/P second-difference, Laplacian and the cross-difference spreading parameters defined below. The associated weights are given by the adjacent upper case letters. The indices ℓ and m assume both integer and half-integer values.

Second-Difference

$$r_{\ell,m} \Rightarrow p_{\ell-1,m+1} + p_{\ell,m} - 2p_{\ell-1/2,m+1/2}$$

$$s_{\ell,m} \Rightarrow p_{\ell+1,m+1} + p_{\ell,m} - 2p_{\ell+1/2,m+1/2}$$

Laplacian

$$t_{\ell,m} \Rightarrow p_{\ell+1/2,m+1/2} + p_{\ell+1/2,m-1/2} + p_{\ell-1/2,m+1/2} + p_{\ell-1/2,m-1/2} - 4p_{\ell,m}$$

Cross-Difference

$$w_{\ell,m} \Rightarrow p_{\ell,m+1} - p_{\ell+1/2,m+1/2} + p_{\ell,m} - p_{\ell-1/2,m+1/2}$$

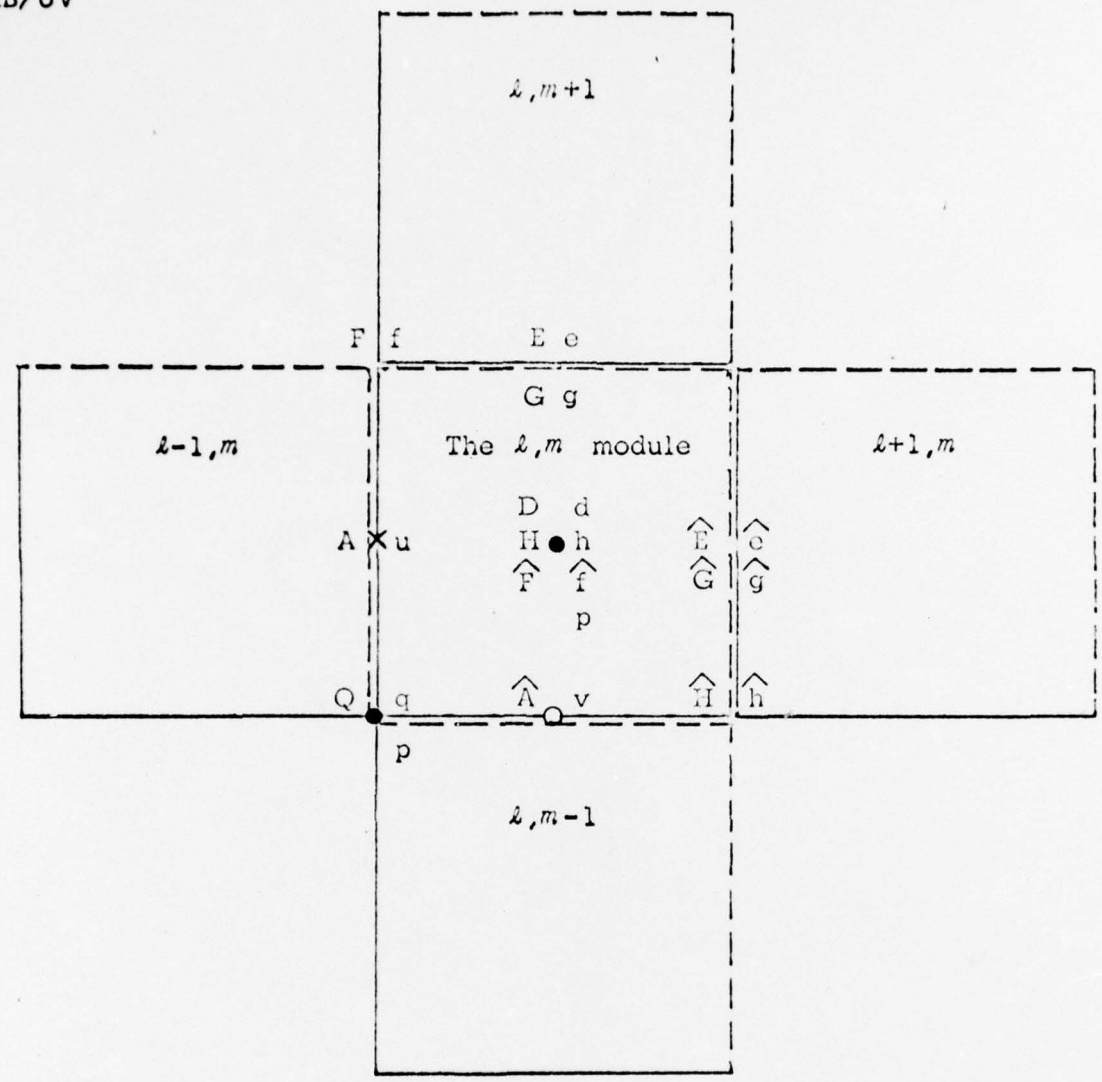

Fig. 7 The arbitrary ℓ,m area module (consisting of one corner point, two sides, and the interior area) and adjoining modules. Reference locations in the module are shown for the u and v components of the wind velocity, and the finite difference parameters:

$$d_{\ell,m} \Rightarrow u_{\ell+1,m} - u_{\ell,m} + v_{\ell,m+1} - v_{\ell,m} ;$$

$$q_{\ell,m} \Rightarrow v_{\ell,m} - v_{\ell-1,m} - u_{\ell,m} + u_{\ell,m-1} ;$$

$$e_{\ell,m} \Rightarrow u_{\ell,m+1} - u_{\ell+1,m} ; \hat{e}_{\ell,m} \Rightarrow v_{\ell,m+1} - v_{\ell+1,m} ;$$

$$f_{\ell,m} \Rightarrow u_{\ell,m+1} - u_{\ell,m} ; \hat{f}_{\ell,m} \Rightarrow v_{\ell,m+1} - v_{\ell,m} ;$$

$$g_{\ell,m} \Rightarrow u_{\ell+1,m+1} - u_{\ell,m} ; \hat{g}_{\ell,m} \Rightarrow v_{\ell+1,m+1} - v_{\ell,m} ;$$

$$h_{\ell,m} \Rightarrow u_{\ell+1,m} - u_{\ell,m} ; \hat{h}_{\ell,m} \Rightarrow v_{\ell+1,m} - v_{\ell,m} .$$

The weights of the values are given by the adjacent upper case letters in the figure. The reference location for the pressure, p, from FIB/P are also shown.

Generalization of the Grid Location, Orientation, Size and Mesh Length

The analyses are done on a polar stereographic, rectangular grid. The grid dimensions are variable up to (MxN) = (63x63)* and the grid mesh length is variable down to the resolution afforded by the density and distribution of wind and pressure reports in the area of application.

A complete specification of the portable grid includes specification of the center of the grid, the rotation relative to the hemispheric grid, the dimensions M and N, and the grid length as a fraction of the standard 63x63 hemispheric grid length (i.e., the grid length factor). Figure 8 shows the standard NH 63x63 grid with the embedded greater Mediterranean regional grid. Table 1 specifies the grid parameters for each. The standard FNWC 63x63 hemispheric grid has a mesh length of 381 km at 60°N. The regional grids used for FIB/SLP-MED [5] and FIB/UV-MED [13] employ a grid length factor of 0.25.

*The total number of grid points on the MxN grid is MxNx2 including the module center grid points. See Fig. 1.

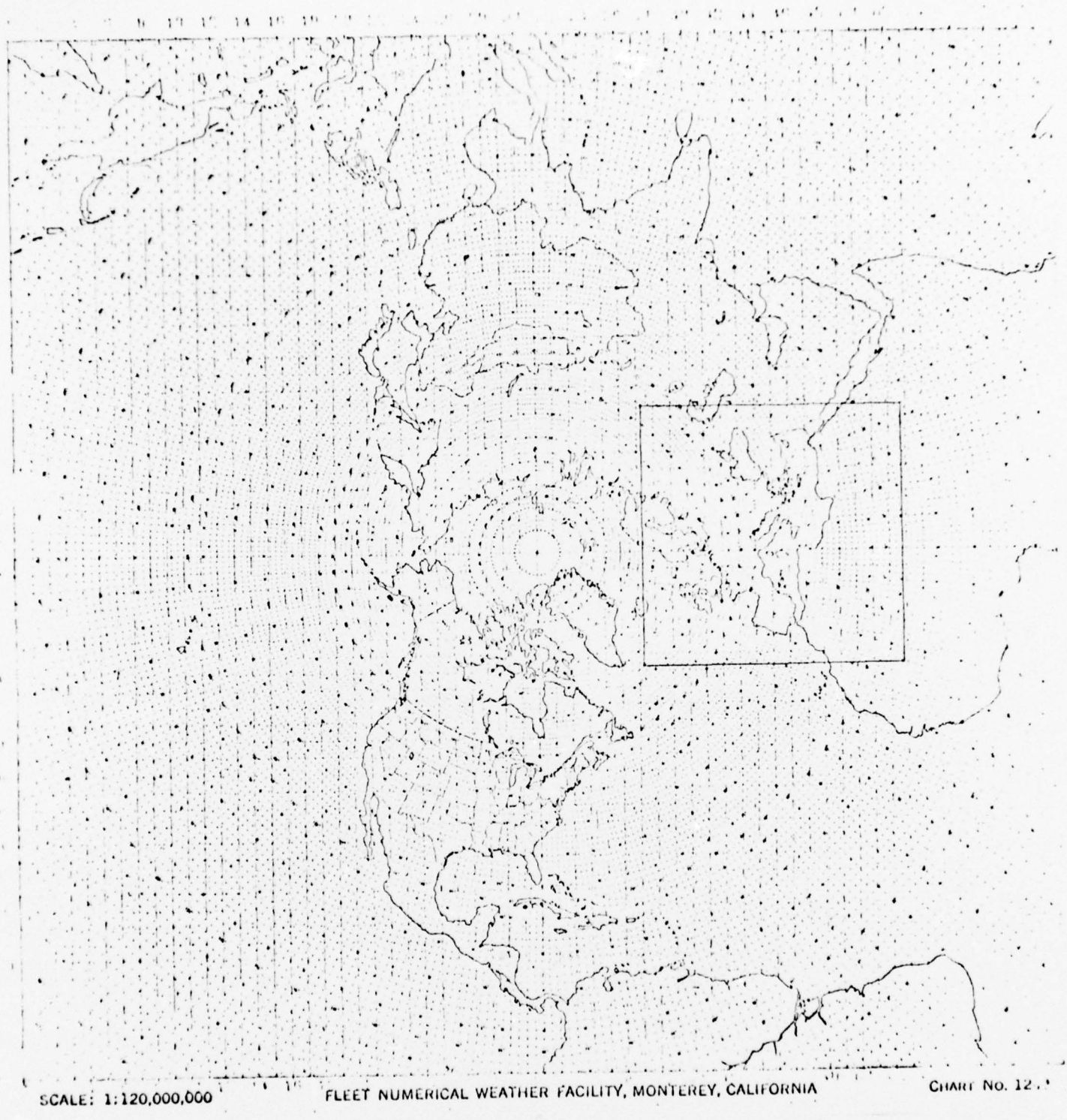

Fig. 8 FNWC Northern Hemisphere, polar stereographic 63x63 grid with embedded greater Mediterranean region indicated.

Table 1

Grid Parameters for the FNWC Northern Hemisphere,
Polar Stereographic 63x63 Grid and the Embedded
Greater Mediterranean Grid

	NH	MED
Center of Grid:	I = 31, J = 31	I = 46, J = 33 (on NH Grid)
Counter-Clockwise Rotation:	0°	0°
MxN:	63x63	63x63
Grid Length Factor:	1.0	.25

Generation of Terrain Parameters

Although the areas of primary interest are over the oceans, it is anticipated that the specific grid encompassing many of these regions will include adjacent land masses and islands. Consequently, the terrain parameters used in the FIB/UV and FIB/P analyses will be required for each new portable grid employed which includes land masses and islands. These parameters are a function of the land topography and the relative percentages of land/sea within the data assembly and reference modules on the grid.

In the FIB/UV analysis, the weight associated with the assembled wind report value in each data assembly module is reduced as a function of the module terrain roughness and land/sea mix--both factors contributing to non-representativeness in the winds as opposed to modules over the open ocean or flat land. In addition, over rough terrain the winds between adjacent grid points will be less strongly correlated than over smooth terrain. Therefore, first-difference and other higher-order spreading weights are reduced as a function of the specific reference module terrain roughness. These formulations are given in more detail in reference [27].

In the FIB/P analysis, the weights associated with the assembled pressure report values are reduced over land as a function of mean assembly module elevation to reflect the uncertainty in the reduction of pressure to sea level. In addition, assembled first-difference weights, which reflect the assimilation of wind reports via the balance relationship, are reduced as a function of mean assembly module elevation and terrain roughness. The first reduction accounts for extrapolation to sea level and the second for decreased correlation between points because of uneven terrain. Also, the weights of higher-order spreading parameters are reduced because of uneven terrain. These formulations are given in more detail in reference [31].

The terrain parameters are presently derived from the FNWC, Northern Hemisphere, high-resolution terrain data available on magnetic tape. The capability to process this raw data into the required terrain parameters for arbitrary reference and data assembly modules has been developed under earlier contract with EPRF [30]. The present capability has been extended to compute all the required terrain parameters necessary for both FIB/UV and FIB/P on the specific grid, to output and catalogue (for future reference) the parameters in the required format to interface with FIB/UV and FIB/P, and to generalize the formulation as a function of grid-mesh length for assimilating the parameters into the analyses.

Grids extending into the Southern Hemisphere will not have the benefit of the terrain parameters since the basic terrain data for the Southern Hemisphere are not currently available. However, the capability to use the terrain parameters will be in the models whenever the Southern Hemisphere data become available.

Generalization of Reevaluation Procedures

In the FIB analysis scheme, three cycles are employed. After the first two cycles, input data weights are reevaluated as a function of the consistency between individual input values and all other information. The weights of individual information elements which are not consistent enter the second and third data assembly cycles with reduced weight according to the degree of disparity. The formulation of this reevaluation is dependent on the resolution of the specific grid, as is the initial weighting of the information elements.

Generalization of Weighting Formulations

Each element of input information in a FIB analysis is weighted according to its reliability (see Figs. 2 through 7 for the specific parameters and associated weights used in FIB/P and FIB/UV). In general, the weights are a function of the grid-mesh length (i.e., resolution) as well as of the quality of the information. In addition, the assembled weights of first-difference pressure values in FIB/P are reduced before blending to reflect the inherent deficiencies in the diagnostic balance relationship used to express reported winds as equivalent pressure differences. This weight reduction is, among other things, grid-mesh-length dependent.

First-Guess Field Preparation

The first-guess field for FIB/UV is prepared directly from the concurrent FIB/P analysis using the diagnostic balance approximation.

The *first-guess pressure field at analysis time* is, in general, a combination of the kinematically extrapolated fields from the previous analysis and a later analysis. When only the previous analysis is available, no combining is necessary. Kinematical extrapolation is performed with geostrophic winds derived from the SR500 field height values.*

This method of computing the first-guess field provides for both forward and backward time continuity in the FIB/P analysis model, and, consequently, in the concurrent FIB/UV analysis.

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*The SR500 field is the actual 500-mb height field with the disturbance scale removed.

Set-up, Start-up and Scheduling Options

The tandem FIB/UV-FIB/P analysis capability available on an "as-required" basis on a portable grid and with provision for backward and forward continuity in time is a powerful and flexible tool. This flexibility requires the consideration and specification of several types of options.

1 -- Set-up options: Before the model is run, certain tasks must be performed. These include specification of the grid parameters and generation of the required terrain parameters as discussed above.

2 -- Start-up options: The model must be run for some specified "start-up" period preceding the "operational-use" period.

3 -- Scheduling options: The actual run times for the analyses must be specified.

Items 2 and 3 above will now be discussed in more detail.

Start-up Options

Each new request to begin a series of analyses will have to specify an operational use period for the analyses. The end points of the operational use period will be given as t_2 and t_3. The operational use interval, then, is $t_3 - t_2 > 0$. The start-up time, t_0, for the model will have to be sufficiently in advance of t_2 so that the effect of the initial conditions introduced by the bootstrap technique have been eliminated. We will indicate this interval for elimination of bootstrap effects by $t_1 - t_0$ where $t_1 < t_2$. In addition, the analyses will be run at fixed intervals, Δt. These relationships are summarized in Fig. 9.

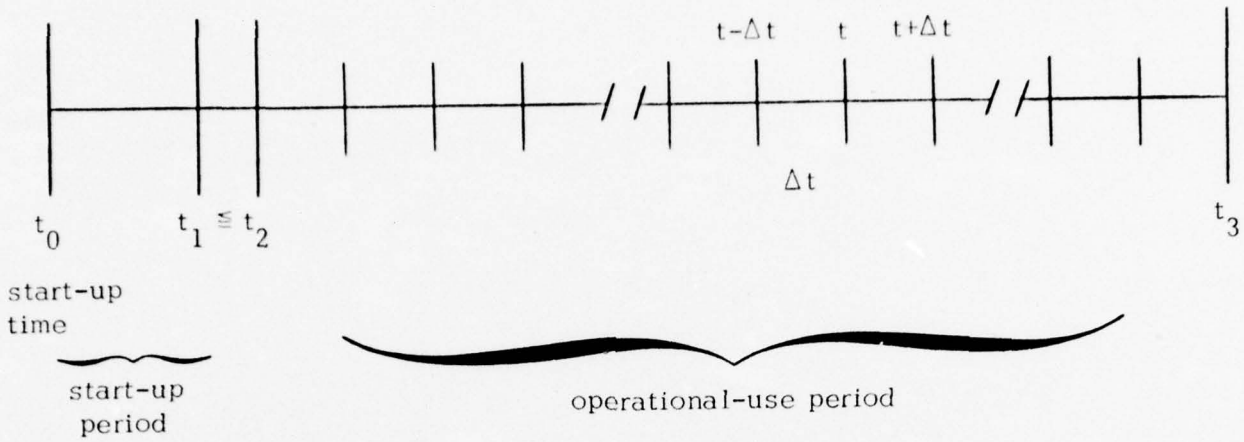

Fig. 9 Schematic of Time Intervals and Relations.

Due to the provision for forward and backward continuity in FIB/P, the start-up period should be short. Also, the existence of a concurrent hemispheric or regional sea-level-pressure analysis into which the portable grid can be embedded will further shorten the start-up period.

Scheduling Considerations

Scheduling the runs into the operational job stream involves many considerations. Among these are the following:

1. Operational:
   a. Requirement to interface with other products (e.g., input to forecast or analysis model).
   b. Requirements to transmit to the field.
2. Data:
   a. Lag time between observations and receipt of observations.
   b. Data-rich vs. data-poor areas and periods.
   c. Degree of updating of analyses to be performed.
3. Time Resolution:
   a. Analysis period.
   b. Extent of forward and backward continuity desired.
4. Processing time:
   a. Amount of computer time available.
   b. Period when computer time available.
   c. Model running time.
5. Historical study vs. operational use.

The scheduling for a specific application will, in general, involve all of the above considerations. No single scheduling will be optimum to meet all possible applications of the model.

3. Supplemental Details FIB/P

3.1 FIB/P Component Operations

1. Set Constants: analysis time, grid definition and identification, and analysis cycle time increment.

2. Read Elevation Data.

   Set to zero if not found or not yet produced.

3. Set Adjustable Constants.

4. Read First-Guess Analyses.

   a. Latest SR500 analysis for kinematic extrapolation

   b. Previous and following FIB/P 125x125 analyses nearest to current time (up to $\pm$9 hrs) { previous time=IAH(1) $<$ 0; following time=IAH(2) $>$ 0

      Use FIB/SLP version if and only if no FIB/P version analyses found.

5. Determine Number of Relaxation Passes.

   NTOTHR = IAH(2) - IAH(1) $\quad |IAH(m)| \leq 9$

   a. If only previous or following analyses found

      NPTY = NTOTHR/2 + 1

   b. If both found

      NPTY = NTOTHR/3 - 1

Ages of First Guesses		NPTY	Number of Passes		
1 found	2 found		Cycle 1	Cycle 2	Cycle 3
6	6,9	$\geq$ 4	15	10	6
	6,6	3	13	9	6
3	3,6	2	10	8	5
	3,3	1	8	7	5

6. Combine with Tropical Field(s).

$W_H = 2 \sin \varphi \qquad (0 < W_H < 1)$

$W_T = 1 - W_H$

φ = latitude
W = weight
H = hemispheric (FIB)
T = tropical (global band)

INTRP used in tropical field.

7. Kinematically Extrapolate Combined Field(s) in 3 hour steps.

   FKX = 0.58 x SR500 geostrophic velocity

   a. Linear interpolation used in moving I,J indices with SR500.

   b. STGNTR(12 point) interpolation uses these I,J values to interpolate in previous and/or following analyses.

8. Combine Extrapolated Fields into First Guess, inversely weighted by time (IAH).

9. Read Observational Data.

   Location, age, type, name, wind, pressure and pressure tendency, ship movement, classification, etc.

   a. ships (hourly)

   b. synops (3 hourly)

   c. airways (hourly)

10. Sort Land Reports north to south.

    Save nearest to 0 hr when duplicates found.

11. Move +3 hr reports back to 0 hr using tendency group (ship and synop) and movement code (ship).

12. Sort Ships by Name.

    Delete same name report if within distance equivalent to 2° latitude and pressure within 2mb or age / 0.

13. Read Artificial ("Bogus") Data.

    a. Forced pressure/wind report

    b. Forced accept or reject of wind or pressure by station name or area

    c. Tropical storm

14. Sort all Data by J (required for ordered assembly).

15. Form Final Data List (4 words/report).

    Determine report weights.

    Convert wind to equivalent pressure difference.

16. Set Accept/Reject Bits in data list from area and name type boguses.

17. Set up Tropical Pressure Fields from TRO Boguses--used for pressure differences only.

18. a. Compute Spreading Parameters from first-guess field

    first deriv weights = W1
    higher deriv weights = W2 (= 0 where undefined)
    Laplacian weights = W2T

    W1 = 0.0001
    W2 = 0.02
    W2T = 0.1

    b. Use "tropical pressure field" only in vicinity of tropical storms (vicinity defined as 2 x 30 kt radius).

19. Get Mean-Square Laplacian Field (symmetrical 5-point mean). Set low order bit only in vicinity of tropical storms.

20. Add Terrain Roughness and Laplacian Variance to higher derivative fields, except in vicinity of tropical storms.

$$\text{added variance} = (CS)\,\sigma^2_{elev} + (CL)\,\overline{L^2} \qquad \begin{matrix} CS = 1.0 \\ CL = 0.6 \end{matrix}$$

21. Get higher deriv (R,S,T,W) portions of a,b,c,α,β,γ,δ,ϵ,R fields and save.

Main Loop Begins

22. Assemble Data.
    a. pressure
    b. 6 first deriv components

23. Add Elevation Variance to Pressure Assembly.
    Added Variance = CEP x $(ELEV)^2$ CEP = 0.001 x $(ft/100)^{-2}$

24. Add Elevation, Terrain Roughness and Balance Approximation Variance to first-difference assemblies.

25. Compute a,b,c,α,β,γ,δ,ϵ,R fields, omitting R,S,T,W terms.

26. Combine Fields with and without R,S,T,W terms.

27. Initialize Fields for Forward Elimination, and Backward Substitution.

28. Compute p* Iteratively by line SOR. Jump to 30 if final cycle.

29. Reevaluate Pressure and Wind Reports, then jump to 22.
    Skip this step if final cycle.

30. Output Analysis, Varian Data List and Reject List.
    Analyses output in 125x125 form (points added from staggered grid)
    and 63x63 form ("center" points omitted).

3.2 FIB/P Component Details

FIB/P Kinematic Extrapolation (Refer to Section 3.1 item 7)

① Zoom steering field to same scale as object field with "center points" omitted. Not required if doing hemispheric analysis.

② Pack I,J starting values into steering field.

③ Extrapolate I,J displacement fields in 3-hr steps. If hemispheric analysis, displacement is damped equatorward from 20° lat.

④ Fill in I,J fields at "center points" using 4-point means.

⑤ Interpolate at I,J's in starting field for all grid points.

⑥ If regional analysis, interpolate in hemispheric field when I,J is closer than "x" grid lengths from edge. Combine the values obtained, weighted by distance from edge:

$$W_{hem} = \frac{x - d}{x} \quad , \quad W_{obj} = 1 - W_{hem} \quad , \quad W_{comb} = W_{hem} + W_{obj}$$

where $0 \leq W_{hem} \leq 1$,

$x = \frac{1.5}{GLF} + 0.5$ (truncated), and

d = fractional number of grid lengths from nearest edge (measured in direction normal to edge).

⑦ Repeat ① through ⑥ if extrapolating backward as well as forward in time. Then combine these, weighted by amount each is off-time.

Note. The above method does not require adding a border region around the regional grid, as previously done.

FIB/P Pressure Assembly (Refer to Section 3.1 item 22a)

Initial report weight = $A_n = 0.5$

(tuned for approximately 67% $\lambda^2 < 1$)

in each assembly cycle:

$$p_n = p_o + p_{rpt} - p_i$$

where

p_n = assembly ("extrapolated") pressure at ref. grid point

p_o = first-guess pressure at ref. grid point

p_{rpt} = reported pressure at station location

p_i = first-guess pressure interpolated at station location
STGNTR 12-point interpolation used

FIB/P First-Difference Assemblies (Refer to Section 3.1 item 22b)

Definition:

GLF = grid length factor $\begin{cases} = 1 & \text{(N. Hem)} \\ = 0.25 & \text{(Med.)} \end{cases}$

$$GLS60 = \left(\frac{GLF}{1 + \sin 60^\circ}\right)$$

$$VWCON = 0.47^* \ (GLS60)^2$$

Computed for Each Wind Observation:

wind variance (wind units):

$$VVEL = 9 + (FF-20)^2/40$$

FF = observed speed (m/sec)

report variance in pressure diff units:

$$VWND = (VWCON)\ [\ (1 + \sin\theta)\ (\sin\theta)\ ]^2\ (VVEL)$$

report weight for assembly

$$B_n = \text{min. of } \left[\frac{1}{VWND}\ ,\ 7.875\right]$$

Six components (b,c,d,e,f,g) are assembled with weight B_n at grid points which are dependent on exact location within module (see p. 27, M-209 Technical Summary).

*The value 0.47 was derived for c,e components but is also used in weighting the b,d,f,g components.

FIB/P First-Difference Variance Additions by Grid Points
(Refer to Section 3.1 item 24)

Variance added by grid point after assembly for:

1. Elevation
2. Terrain Roughness
3. Balance Approximation

1. Elevation Variance = (CE) $(\overline{E})$

   $\overline{E}$ = mean modal elevation in module area

   CE = 0.2 for $\overline{E}$ in ft/100

2. Terrain Roughness Variance = (CS) (σ^2_E)

   σ_E = std dev of modal elevations in module

   CS = 1.0 for σ^2_E in ft/100

3. Balance Approx. Variance:

   $\sigma^2_{bal} = 1.0$

FIB/P Pressure Reevaluation (Refer to Section 3.1 item 30a)

$$\lambda_p^2 = \frac{\left(p_n - p^*\right)^2}{\sigma_c^2 + \sigma_g^2}$$

where

p_n = report pressure, extrapolated

p^* = analyzed pressure at grid point

σ_c^2 = class variance

σ_g^2 = gradient variance

= mean diagonal first-difference squared / F_g

F_g = GSQF $(GLF)^2$ GSQF = 6

σ_g^2 limit = 4

Modification for low pressure:

λ_p^2 is multiplied by:

$$F_p = \frac{p^* - 920}{100} \text{ (mb)} \qquad \left[0.4 \leq F_p \leq 1.0\right]$$

Reevaluated weight:

$$ANR = AN/\left(1 + \left(\lambda_p - 1\right)^2\right)$$

ANR = reevaluated weight

AN = class weight

FIB/P First-Difference Reevaluation (Refer to Section 3.1 item 30b)

$$\lambda_d^2 = \frac{(c_n - c^*)^2 + (e_n - e^*)^2}{1/B_n}$$

where

B_n = first cycle assembly weight (b,c,d,e,f,g components)

c,e = J,I direction first differences

$\lambda_d^2 = \lambda^2$ for first difference

Modified λ_d^2 for high wind speed:

λ_d^2 is multiplied by:

$$F_d = \frac{3.5}{\text{MIN}\left[\left(\mu^{*2} + \nu^{*2}\right)^{1/2}, \left(\mu_r^2 + \nu_n^2\right)^{1/2}\right]}$$

Second Test:

$$\lambda_d^2(2) = \frac{(c_n - c^*)^2 + (e_n - e^*)^2}{(c_n + c^*)^2 + (e_n + e^*)^2 + \text{XLWGK}} \cdot \text{FXLWG}$$

where

XLMTW = 6

FXLWG = XLMTW/0.7

XLWGK = $0.4(\text{GLF})^2$

Reject if λ_d^2 or $\lambda_d^2(2) >$ XLMTW .

For reports kept:

<u>reduce weight</u> if $\lambda_d^2 > 1$:

$$B_r = \frac{B_n}{1 + (\lambda - 1)^2}$$

4. Supplemental Details FIB/UV

4.1 FIB/UV Component Operations

1. Define portable grid information and adjustable constants
   - Center I,J (in 63x63 grid)
   - Scale factor (fraction of 63x63 grid length)
   - Grid dimensions
2. Read elevation data
   - If data not available, zero values are assumed (correct over ocean areas).
3. Read current pressure field and diagnose U,V.
4. Compute K,R weighted by fraction of land/sea coverage upstream.
5. Add terrain roughness variance to first-guess variance fields.
6. Get first-guess divergence and vorticity from U_o, V_o.
7. Get R and S terms from first-guess fields, omitting weights to be assembled.
8. Read data and convert to internal format. Eliminate duplicates.
   a. K,R computed explicitly from observed wind and pressure gradient for each observation.
   b. "Special station" list search for matches. Observations in special list are flagged and given listed weight.
9. Main loop begins--assemble data. First pass assembly weights reduced on basis of first-guess U,V components, when large discrepancies are computed. Land reports assembled first, then ships. Elevation variances added to land assembly only.

10. Add assembled weights to R,S terms.

11. Set up matrices for line SOR.

12. Get blended U,V fields by line relaxation. Go to #15 if first pass through main loop.

13. U,V values at staggered grid points converted to standard grid points by cubic interpolation.

14. Output U,V and direction, speed fields. Go to #16.

15. Reevaluate U,V components. List ship reports and any stations in special list. List includes name, location, nearest grid point, reported and analyzed direction and speed, vector difference, K and R computed from observed wind and pressure gradient, beginning and reevaluated weights, λ^2 and observed and nearest grid point U and V components. A special notation is printed for special stations and for rejects. Go to #9 if first cycle.

16. List λ^2 summaries--frequency distribution of λ^2 values by report type, and mean and median λ^2 and reported wind speed by classes of analyzed wind speed for ship reports.

4.2 FIB/UV Adjustable Constants

Following constants affect the grid placement and are defined in the program call:

I,J (integer) define the center point relative to the standard grid

N (integer) defines the grid dimension

G (real) defines the mesh length as a fraction of the standard grid.

The above values are order independent. Example of call:

```
FIBUV(6,3,G=0.25,I=25,J=41,N=29)
```

This first parameter (6) defines the observation time as 6 hours after the current time in the computer. The second parameter (3) indicates a portable grid application. Other numbers are used for fixed areas. G=0.25 indicates a 1/4 mesh grid. I=25,J=41 indicates the grid is centered at this grid point in the 63x63 grid numbered from 0 to 62. N=29 indicates the dimensions of the grid are 29x29.

The following constants are defined in the program under the heading "Adjustable Constants"

XKPL	land value of frictional constant K
XKPS	sea value of frictional constant K
RLAND	land value of frictional constant R
RSEA	sea value of frictional constant R
VLW	class variance of land wind report
VSW	class variance of sea wind report
AWT	first-guess A weight (before addition of other variances)
DWT	first-guess D weight
QWT	first-guess Q weight
EFWT	first-guess E and F weight
GHWT	first-guess G and H weight
ELSCL	first- and second-difference elevation roughness scaling factor
SCLRF	U,V assembly elevation roughness scaling factor
VLSMAX	maximum variance addition to U,V assembly due to land/sea effect

5. Running Procedure

The following control cards are required to run a portable grid application. The example shown is for the current time on a 1/4 mesh 19x19 grid centered at [31,13]:

```
JOBCARD,EC300,T300.
APLIB(,*FIBP,*FIBUV)
RFE,300.
FIBP(0,3,N=19,I=31,J=13,F=0.25)
FIBUV(0,3,N=19,I=31,J=13,F=0.25)
```

The first parameter in the call indicates the number of hours before (-) or after (+) the current machine data-time group. The second parameter is always 3 to indicate a portable grid application. The other parameters are respectively, the grid dimensions, the I and J location of the center in the standard grid system, and the mesh size fraction.

The above application requires approximately 19 sec central processor time on the CDC 6500 for FIB/P and 15 sec for FIB/UV.

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APPENDIX A

Solution of the Blending System of Equations by Line SOR

The system of blending equations to be solved may be written as

$$\underset{\approx}{C}\ \underset{\sim}{P^*} = \underset{\sim}{R}$$

where the ordering of the elements within the vectors $\underset{\sim}{P^*}$ and $\underset{\sim}{R}$ will be that of increasing values of k within increasing values of n (i.e., the k,n grid-point notation introduced in Section 1.3 and shown schematically in Fig. 3). This permits us to partition $\underset{\sim}{P^*}$ and $\underset{\sim}{R}$ into subvectors $\underset{\sim}{P^*}_n$ and $\underset{\sim}{R}_n$, respectively, where each subvector corresponds to a row of unknown pressure elements or forcing functions. These vectors are summarized as follows:

$$\underset{\sim}{P}^{*} = \begin{bmatrix} \\ \underset{\sim}{P}^{*}_{1} \\ \underset{\sim}{P}^{*}_{2} \\ \vdots \\ \underset{\sim}{P}^{*}_{n} \\ \vdots \\ \underset{\sim}{P}^{*}_{M-1} \\ \underset{\sim}{P}^{*}_{M} \\ \\ \end{bmatrix} \qquad \text{where} \qquad \underset{\sim}{P}^{*}_{n} = \begin{bmatrix} p^{*}_{1,n} \\ p^{*}_{2,n} \\ \vdots \\ p^{*}_{k-1,n} \\ p^{*}_{k,n} \\ p^{*}_{k+1,n} \\ \vdots \\ p^{*}_{K-1,n} \\ p^{*}_{K,n} \end{bmatrix}$$

and

$$\underset{\sim}{R} = \begin{bmatrix} \\ \underset{\sim}{R}_{1} \\ \underset{\sim}{R}_{2} \\ \vdots \\ \underset{\sim}{R}_{n} \\ \vdots \\ \underset{\sim}{R}_{M-1} \\ \underset{\sim}{R}_{M} \\ \\ \end{bmatrix} \qquad \text{where} \qquad \underset{\sim}{R}_{r.} = \begin{bmatrix} \hat{R}_{1,n} \\ \hat{R}_{2,n} \\ \vdots \\ \hat{R}_{k-1,n} \\ \hat{R}_{k,n} \\ \hat{R}_{k+1,n} \\ \vdots \\ \hat{R}_{K-1,n} \\ \hat{R}_{K,n} \end{bmatrix}$$

Referring to the 13-point stencil for the blending system of equations (Fig. 7), it can be seen that when the center point (ℓ, m) of the stencil is the corner point of a grid module (integer values of ℓ and m), the subvector $\underset{\sim}{P}^*_{n+1}$ will have three non-zero coefficients while subvectors $\underset{\sim}{P}^*_n$ and $\underset{\sim}{P}^*_{n-1}$ will each have five non-zero coefficients. However, when the center point (ℓ, m) of the stencil is the center point of a grid module (half-integer values of ℓ and m), the subvector $\underset{\sim}{P}^*_{n-1}$ will have three non-zero coefficients while subvectors $\underset{\sim}{P}^*_n$ and $\underset{\sim}{P}^*_{n+1}$ will each have five non-zero coefficients.

An equivalent partitioned form of Eq. (5) is then

$$\begin{bmatrix} \underset{\approx}{A}_1 & \underset{\approx}{B}_1 & & & & & \\ \underset{\approx}{B}_1^T & \underset{\approx}{A}_2 & \underset{\approx}{B}_2 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & \underset{\approx}{B}_{n-1}^T & \underset{\approx}{A}_n & \underset{\approx}{B}_n & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \underset{\approx}{B}_{M-2}^T & \underset{\approx}{A}_{M-1} & \underset{\approx}{B}_{M-1} \\ & & & & & \underset{\approx}{B}_{M-1}^T & \underset{\approx}{A}_M \end{bmatrix} \begin{bmatrix} \underset{\sim}{P}^*_1 \\ \underset{\sim}{P}^*_2 \\ \vdots \\ \underset{\sim}{P}^*_n \\ \vdots \\ \underset{\sim}{P}^*_{M-1} \\ \underset{\sim}{P}^*_M \end{bmatrix} = \begin{bmatrix} \underset{\sim}{R}_1 \\ \underset{\sim}{R}_2 \\ \vdots \\ \underset{\sim}{R}_n \\ \vdots \\ \underset{\sim}{R}_{M-1} \\ \underset{\sim}{R}_M \end{bmatrix}$$

where $\underset{\approx}{C}$ has been rewritten as a block tridiagonal matrix. The elements of $\underset{\approx}{A}_n$ come from the middle row of the stencil, the elements of $\underset{\approx}{B}_n$ come from the top row of the stencil, and the elements of $\underset{\approx}{B}_{n-1}^T$ come from the bottom row of the stencil. Both the $\underset{\approx}{A}_n$ and $\underset{\approx}{B}_n$ submatrices are KxK and pentadiagonal. In addition, the $\underset{\approx}{A}_n$ are symmetric and positive-definite.

The matrix $\underset{\approx}{C}$ may be written as

$$\underset{\approx}{C} = \underset{\approx}{A} + \underset{\approx}{B} \tag{A1}$$

where

$$\underset{\approx}{A} = \text{diag}\left(\underset{\approx}{A}_1, \underset{\approx}{A}_2, \ldots, \underset{\approx}{A}_M\right) \tag{A2}$$

and $\underset{\approx}{B}$ is given by

$$\begin{bmatrix} \underset{\approx}{O} & \underset{\approx}{B}_1 & & & & \\ \underset{\approx}{B}_1^T & \underset{\approx}{O} & \underset{\approx}{B}_2 & & & \\ & \underset{\approx}{B}_2^T & \underset{\approx}{O} & & & \\ & & & \ddots & & \\ & & & & \underset{\approx}{O} & \underset{\approx}{B}_{M-1} \\ & & & & \underset{\approx}{B}_{M-1}^T & \underset{\approx}{O} \end{bmatrix} .$$

The blending system equations may then be rewritten as

$$\left(\underset{\approx}{A} + \underset{\approx}{B}\right) \underset{\sim}{P^*} = \underset{\sim}{R} \qquad \text{(A3)}$$

or as

$$\underset{\approx}{A}_n \underset{\sim}{P^*}_n + \underset{\approx}{B}_{n-1}^T \underset{\sim}{P^*}_{n-1} + \underset{\approx}{B}_n \underset{\sim}{P^*}_{n+1} = \underset{\sim}{R}_n \qquad n = 1,2,\ldots,M \qquad \text{(A4)}$$

where

$$\underset{\approx}{B}_o \equiv \underset{\approx}{B}_M \equiv \underset{\approx}{O} .$$

Equation (A4) may be solved for $\underset{\sim}{P}^*_n$ as

$$\underset{\sim}{P}^*_n = \underset{\approx}{A}_n^{-1} \left[\underset{\sim}{R}_n - \underset{\approx}{B}^T_{n-1} \underset{\sim}{P}^*_{n-1} - \underset{\approx}{B}_n \underset{\sim}{P}^*_{n+1} \right] . \quad \text{(A5)}$$

The solution by line relaxation is defined by

$$\left(\underset{\sim}{P}^*_n \right)^{(r+1)} = \left(\underset{\sim}{P}^*_n \right)^{(r)} + \left[\underset{\sim}{P}^{*\,(r+1/2)}_n - \left(\underset{\sim}{P}^*_n \right)^{(r)} \right] \omega \quad , \quad \text{(A6)}$$

where the superscript (r) refers to pass number and where

$$\left(\underset{\sim}{P}^*_n \right)^{(r+1/2)} = \left(\underset{\approx}{A}_n \right)^{-1} \left[\underset{\sim}{R}_n - \underset{\approx}{B}^T_{n-1} \left(\underset{\sim}{P}^*_{n-1} \right)^{(r+1)}_{(r)} - \underset{\approx}{B}_n \left(\underset{\sim}{P}^*_{n+1} \right)^{(r+1)}_{(r)} \right] . \quad \text{(A7)}$$

The upper superscript (r) applies to the term in brackets for the first half of each pass in which either the even or odd numbered n (hereafter referred to as red lines) are solved for. The lower superscript (r+1) applies to the term in brackets for the second half of each pass in which the remaining (either odd or even) n (hereafter referred to as black lines) are solved for. The method converges for $0 < \omega < 2$ where $1 < \omega < 2$ implies overrelaxation. The inversion specified by Eq. (A7) is obtained explicitly by a triangular decomposition of the $\underset{\approx}{A}_n$ matrices into the product of an upper and a lower triangular matrix, followed by Gaussian elimination.

We will now present the details of the method of solution by line relaxation.

If we let

$$\underset{\sim}{X}_n \equiv \left(\underset{\sim}{P}^*_n \right)^{(r+1/2)} \quad \text{(A8)}$$

and

$$\underset{\sim}{\Gamma}_n = \left[\underset{\sim}{R}_n - \underset{\approx}{B}_{n-1}^T \left(\underset{\sim}{P}^*_{n-1} \right)^{(r)}_{(r+1)} - \underset{\approx}{B}_n \left(\underset{\sim}{P}^*_{n+1} \right)^{(r)}_{(r+1)} \right] \tag{A9}$$

then Eq. (A7) may be written in the equivalent form

$$\underset{\approx}{A}_n \underset{\sim}{X}_n = \underset{\sim}{\Gamma}_n \quad . \tag{A10}$$

Since each $\underset{\approx}{A}_n$ is symmetric and positive definite, it can be decomposed uniquely into $\underset{\approx}{L}_n \underset{\approx}{U}_n$, where $\underset{\approx}{L}_n = \underset{\approx}{U}_n^T$ is a lower triangular matrix with positive diagonal elements. Equation (A10) is rewritten as

$$\underset{\approx}{L}_n \underset{\approx}{U}_n \underset{\sim}{X}_n = \underset{\sim}{\Gamma}_n \quad . \tag{A11}$$

The solution for $\underset{\sim}{X}_n$ can then be obtained by first solving the triangular system

$$\underset{\approx}{L}_n \underset{\sim}{Z}_n = \underset{\sim}{\Gamma}_n \tag{A12}$$

for $\underset{\sim}{Z}_n$ and subsequently solving

$$\underset{\approx}{U}_n \underset{\sim}{X}_n = \underset{\sim}{Z}_n \tag{A13}$$

for $\underset{\sim}{X}_n$. The elements of $\underset{\sim}{Z}_n$ are easily obtained from Eq. (A12) in the order $Z_{1,n}, Z_{2,n}, \ldots, Z_{K,n}$ since the first equation involves only $Z_{1,n}$, the

second equation involves $Z_{1,n}$ and $Z_{2,n}$, and so on. Then the elements of $\underset{\sim}{X}_n$ are similarly obtained from Eq. (A13) in the order $X_{K,n}, X_{K-1,n}, \ldots, X_{1,n}$. These steps--obtaining the intermediate solution, $\underset{\sim}{Z}_n$, from Eq. (A12) and obtaining the final solution, $\underset{\sim}{X}_n$, from Eq. (A13)--are referred to as forward elimination and backward substitution, respectively.

The elements of the symmetric, pentadiagonal matrix $\underset{\approx}{A}_n$ will be given by

$$\underset{\approx}{A}_n = \begin{bmatrix} a_{1,n} & b_{1,n} & c_{1,n} & & & & \\ b_{1,n} & a_{2,n} & b_{2,n} & & & & \\ c_{1,n} & b_{2,n} & a_{3,n} & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & a_{K-2,n} & b_{K-2,n} & c_{K-2,n} \\ & & & & b_{K-2,n} & a_{K-1,n} & b_{K-1,n} \\ & & & & c_{K-2,n} & b_{K-1,n} & a_{K,n} \end{bmatrix} .$$

The elements of the $\underset{\approx}{B}_n$ matrix will be given by

$$\underset{\approx}{B}_n = \begin{bmatrix} \alpha_{1,n} & \beta_{1,n} & \gamma_{1,n} & 0 & & & \\ \delta_{1,n} & \alpha_{2,n} & \beta_{2,n} & \gamma_{2,n} & & & \\ \epsilon_{1,n} & \delta_{2,n} & \alpha_{3,n} & \beta_{3,n} & & & \\ 0 & \epsilon_{2,n} & \delta_{3,n} & \alpha_{4,n} & \ddots & & \\ & & & & \ddots & & \\ & & & & & \alpha_{K-3,n} & \beta_{K-3,n} & \gamma_{K-3,n} & 0 \\ & & & & & \delta_{K-3,n} & \alpha_{K-2,n} & \beta_{K-2,n} & \gamma_{K-2,n} \\ & & & & & \epsilon_{K-3,n} & \delta_{K-2,n} & \alpha_{K-1,n} & \beta_{K-1,n} \\ & & & & & 0 & \epsilon_{K-2,n} & \delta_{K-1,n} & \alpha_{K,n} \end{bmatrix}$$

The matrix $\underset{\approx}{L}_n$ of the triangular decomposition is given as

$$\underset{\approx}{L}_n = \begin{bmatrix} d_{1,n} & 0 & 0 & & & \\ e_{1,n} & d_{2,n} & 0 & & & \\ f_{1,n} & e_{2,n} & d_{3,n} & & & \\ & & \ddots & & & \\ & & & \ddots & & \\ & & & & d_{K-2,n} & 0 & 0 \\ & & & & e_{K-2,n} & d_{K-1,n} & 0 \\ & & & & f_{K-2,n} & e_{K-1,n} & d_{K,n} \end{bmatrix} .$$

The elements of the upper triangular matrix are specified by $\underset{\approx}{U}_n = \underset{\approx}{L}_n^T$.

Equating the individual elements of $\underset{\approx}{A}_n$ to the corresponding elements of the product $\underset{\approx}{L}_n \underset{\approx}{U}_n$ and solving for the elements $d_{1,n}$, $e_{1,n}$, $d_{2,n}$, $f_{1,n}$, $e_{2,n}$, $d_{3,n}$, etc., the following relationships result:

<u>initialize</u>:

$$d_{1,n} = \left(a_{1,n}\right)^{1/2}$$

$$e_{1,n} = b_{1,n}/d_{1,n}$$

$$d_{2,n} = \left(a_{2,n} - e_{1,n}^2\right)^{1/2}$$

<u>iterate</u>: $k = 1,2,\ldots,K-2$

$$f_{k,n} = c_{k,n}/d_{k,n}$$

$$e_{k+1,n} = \left(b_{k+1,n} - e_{k,n}\, f_{k,n}\right)/d_{k+1,n}$$

$$d_{k+2,n} = \left(a_{k+2,n} - e_{k+1,n}^2 - f_{k,n}^2\right)^{1/2} \quad . \qquad \text{(A14)}$$

Having determined the elements of $\underset{\approx}{L}$ we can solve explicitly for $\underset{\sim}{X}_n$ in Eq. (A10) by employing forward elimination to solve for $\underset{\sim}{Z}_n$ in Eq. (A12) and backward substitution to solve for $\underset{\sim}{X}_n$ in Eq. (A13).

The forward elimination proceeds as follows:

<u>initialize</u>:

$$Z_{1,n} = F_{1,n}/d_{1,n}$$

$$Z_{2,n} = \frac{F_{2,n} - e_{1,n} Z_{1,n}}{d_{2,n}}$$

<u>iterate</u>: $k = 3,\ldots,K$

$$Z_{k,n} = \frac{F_{k,n} - e_{k-1,n} Z_{k-1,n} - f_{k-2,n} Z_{k-2,n}}{d_{k,n}} \quad . \qquad \text{(A15)}$$

The backward substitution then follows:

$$\underline{\text{initialize}}: \quad X_{K,n} = Z_{K,n}/d_{K,n}$$

$$X_{K-1,n} = \frac{Z_{K-1,n} - e_{K-1,n} X_{K,n}}{d_{K-1,n}}$$

$$\underline{\text{iterate}}: \quad k = K-2, K-3, \ldots, 1$$

$$X_{k,n} = \frac{Z_{k,n} - e_{k,n} X_{k+1,n} - f_{k,n} X_{k+2,n}}{d_{k,n}} \quad . \tag{A16}$$

The calculation of each element of $Z_{k,n}$ of $\underset{\sim}{Z}_n$ in the forward elimination and of each element $X_{k,n}$ of $\underset{\sim}{X}_n$ in the backward substitution involves division by the diagonal element $d_{k,n}$. Since the system (A10) is to be solved many times with the same coefficient matrix $\underset{\approx}{A}_n$ but with different forcing vectors, $\underset{\sim}{F}_n$, it would save computation time if we could transform Eq. (A10) so that the resulting diagonal elements are all unity.

This transformation can be accomplished by use of the matrices

$$\underset{\approx}{D}_n = \text{diag} \; \underset{\approx}{L}_n = \begin{bmatrix} d_{1,n} & & \\ & \ddots & \\ & & d_{K,n} \end{bmatrix}$$

and

$$\underset{\approx}{D}_n^{-1} = \begin{bmatrix} 1/d_{1,n} & & \\ & \ddots & \\ & & 1/d_{K,n} \end{bmatrix} .$$

Making the transformations

$$\underset{\approx}{\bar{A}}_n = \underset{\approx}{D}_n^{-1} \underset{\approx}{A}_n \underset{\approx}{D}_n^{-1} , \tag{A17}$$

$$\underset{\sim}{\bar{X}}_n = \underset{\approx}{D}_n \underset{\sim}{X}_n \tag{A18}$$

and

$$\underset{\sim}{\bar{F}}_n = \underset{\approx}{D}_n^{-1} \underset{\sim}{F}_n , \tag{A19}$$

equation (A10) becomes

$$\underset{\approx}{\bar{A}}_n \underset{\sim}{\bar{X}}_n = \underset{\sim}{\bar{F}}_n . \tag{A20}$$

These transformations produce only a rescaling of the $\underset{\approx}{A}_n$, $\underset{\sim}{X}_n$ and $\underset{\sim}{F}_n$ without affecting the character of the coefficient matrices; i.e., $\underset{\approx}{\bar{A}}_n$ is KxK, symmetric, positive definite and pentadiagonal.

The triangular decomposition is now written

$$\underset{\approx}{\bar{A}}_n = \underset{\approx}{\bar{L}}_n \underset{\approx}{\bar{U}}_n \tag{A21}$$

where

$$\underset{\approx}{\bar{L}}_n = \underset{\approx}{D}_n^{-1} \underset{\approx}{L}_n \tag{A22}$$

and

$$\underset{\approx}{\bar{U}}_n = \underset{\approx}{U}_n \underset{\approx}{D}_n^{-1} \quad . \tag{A23}$$

Both $\underset{\approx}{\bar{L}}_n$ and $\underset{\approx}{\bar{U}}_n$ have 1's on the main diagonals and

$$\underset{\approx}{\bar{U}}_n^T = \underset{\approx}{\bar{L}}_n \quad . \tag{A24}$$

The elements of $\underset{\approx}{\bar{A}}_n$ may be written directly from Eq. (A17) as

$$\bar{a}_{k,n} = a_{k,n}/d_{k,n}^2 \quad , \quad k = 1,\ldots,K$$

$$\bar{b}_{k,n} = b_{k,n}/d_{k,n}\, d_{k+1,n} \quad , \quad k = 1,\ldots,K-1$$

$$\bar{c}_{k,n} = c_{k,n}/d_{k,n}\, d_{k+2,n} \quad , \quad k = 1,\ldots,K-2 \tag{A25}$$

where

$$\bar{a}_{1,n} = a_{1,n}/d_{1,n}^2 = 1$$

from Eq. (A14).

The elements of $\underset{\approx}{\bar{L}}_n$ may be obtained directly from Eq. (A22) as

$$\begin{aligned}
\bar{d}_{k,n} &= 1 && k = 1,\ldots,K \\
\bar{e}_{k,n} &= \frac{e_{k,n}}{d_{k+1,n}} && k = 1,\ldots,K-1 \\
\bar{f}_{k,n} &= \frac{f_{k,n}}{d_{k+2,n}} && k = 1,\ldots,K-2 \quad . \qquad \text{(A26)}
\end{aligned}$$

Given the $\underset{\approx}{A}_n$ matrix, it is necessary to compute and save only the $d_{k,n}$ elements for the transformations (A18) and (A19) and the $\bar{e}_{k,n}$ and $\bar{f}_{k,n}$ elements for the forward elimination and backward substitution to obtain $\underset{\sim}{\bar{X}}_n$. These elements are obtained as follows:

<u>initialize</u>:

$$\begin{aligned}
d_{1,n} &= \left(a_{1,n}\right)^{1/2} \\
e_{1,n} &= b_{1,n}/d_{1,n} \\
d_{2,n} &= \left(a_{2,n} - e_{1,n}^2\right)^{1/2} \\
\bar{e}_{1,n} &= e_{1,n}/d_{2,n}
\end{aligned}$$

<u>iterate</u>:

$$\begin{aligned}
k &= 1,2,\ldots,K-2 \\
f_{k,n} &= c_{k,n}/d_{k,n} \\
e_{k+1,n} &= \left(b_{k+1,n} - e_{k,n}\, f_{k,n}\right)/d_{k+1,n} \\
d_{k+2,n} &= \left(a_{k+2,n} - e_{k+1,n}^2 - f_{k,n}^2\right)^{1/2} \\
\bar{f}_{k,n} &= f_{k,n}/d_{k+2,n} \\
\bar{e}_{k+1,n} &= e_{k+1,n}/d_{k+2,n} \quad . \qquad \text{(A27)}
\end{aligned}$$

The $d_{k,n}$, $\bar{c}_{k,n}$ and $\bar{f}_{k,n}$ elements may be stored over the $a_{k,n}$, $b_{k,n}$ and $c_{k,n}$ elements, respectively, as they are computed.

The transformed system is solved for $\bar{\underset{\sim}{X}}_n$ by using forward elimination to solve for $\bar{\underset{\sim}{Z}}_n$ of

$$\bar{\underset{\approx}{L}}_n \, \bar{\underset{\sim}{Z}}_n = \bar{\underset{\sim}{F}}_n \qquad \text{(A28)}^*$$

and backward substitution to solve for $\bar{\underset{\sim}{X}}_n$ of

$$\bar{\underset{\approx}{U}}_n \, \bar{\underset{\sim}{X}}_n = \bar{\underset{\sim}{Z}}_n \quad . \qquad \text{(A29)}$$

The forward elimination proceeds as follows:

<u>initialize</u>:

$$\bar{Z}_{1,n} = \bar{F}_{1,n}$$

$$\bar{Z}_{2,n} = \bar{F}_{2,n} - \bar{e}_{1,n} \bar{Z}_{1,n} \qquad \text{(A30)}$$

<u>iterate</u>: $k = 3,\ldots,K$

$$\bar{Z}_{k,n} = \bar{F}_{k,n} - \bar{e}_{k-1,n} \bar{Z}_{k-1,n} - \bar{f}_{k-2,n} \bar{Z}_{k-2,n} \; . \qquad \text{(A31)}$$

The backward substitution then follows:

<u>initialize</u>:

$$\bar{X}_{K,n} = \bar{Z}_{K,n}$$

$$\bar{X}_{K-1,n} = \bar{Z}_{K-1,n} - \bar{e}_{K-1,n} \bar{X}_{K,n}$$

<u>iterate</u>: $k = K-2, K-3, \ldots, 1$

$$\bar{X}_{k,n} = \bar{Z}_{k,n} - \bar{e}_{k,n} \bar{X}_{k+1,n} - \bar{f}_{k,n} \bar{X}_{k+2,n} \; . \qquad \text{(A32)}$$

---

*From Eqs. (A19) and (A22) we see that $\bar{\underset{\sim}{Z}}_n$ is equivalent to $\underset{\sim}{Z}_n$.

The reverse transform of Eq. (A18); i.e., $\underset{\sim}{X}_n = \underset{\sim}{D}_n^{-1} \overline{\underset{\sim}{X}}_n$, could now be used to obtain $\underset{\sim}{X}_n$.

The preceding discussion has indicated how to solve Eq. (A7) or its equivalent form, Eq. (A10), by Gauss elimination. Since this is only one of M equations of the same form, which must be solved for each complete pass of the relaxation scheme, we will indicate how the entire blending system, given by Eq. (5), is transformed as the initial step in the relaxation procedure so that line relaxation may be used.

The transformations (A17), (A18) and (A19) need to be made only once in solving the blending system of equations given by (5) or its equivalent partitioned forms. Therefore, we will proceed by defining matrix $\underset{\approx}{D}$ as

$$\underset{\approx}{D} = \text{diag}\left(\underset{\approx}{D}_n\right) \tag{A33}$$

and its inverse $\underset{\approx}{D}^{-1}$ by

$$\underset{\approx}{D}^{-1} = \text{diag}\left(\underset{\approx}{D}_n^{-1}\right) \quad . \tag{A34}$$

Then (A3) may be transformed as

$$\underset{\approx}{D}^{-1}\left(\underset{\approx}{A} + \underset{\approx}{B}\right)\underset{\approx}{D}^{-1}\,\underset{\approx}{D}\,\underset{\sim}{P}^{*} = \underset{\approx}{D}^{-1}\,\underset{\sim}{R} \quad . \tag{A35}$$

Let

$$\overline{\underset{\sim}{R}} = \underset{\approx}{D}^{-1}\,\underset{\sim}{R} \quad , \tag{A36}$$

$$\overline{\underset{\sim}{P}}^{*} = \underset{\approx}{D}\,\underset{\sim}{P}^{*} \quad , \tag{A37}$$

$$\overline{\underset{\approx}{B}} = \underset{\approx}{D}^{-1}\,\underset{\approx}{B}\,\underset{\approx}{D}^{-1} \tag{A38}$$

and

$$\bar{\underset{\approx}{A}} \equiv \underset{\approx}{D}^{-1} \underset{\approx}{A} \underset{\approx}{D}^{-1} \quad . \tag{A39}$$

Equation (A35) may then be rewritten

$$\bar{\underset{\approx}{A}} \; \bar{\underset{\sim}{P}}^{*} + \bar{\underset{\approx}{B}} \; \bar{\underset{\sim}{P}}^{*} = \bar{\underset{\sim}{R}} \tag{A40}$$

or as

$$\bar{\underset{\approx}{A}}_n \bar{\underset{\sim}{P}}^{*}_n + \bar{\underset{\approx}{B}}^{T}_{n-1} \bar{\underset{\sim}{P}}^{*}_{n-1} + \bar{\underset{\approx}{B}}_n \bar{\underset{\sim}{P}}^{*}_{n+1} = \bar{\underset{\sim}{R}}_n \qquad n = 1,2,\ldots,M \quad . \tag{A41}$$

The solution by line relaxation for this transformed system is defined by

$$\left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r+1)} = \left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r)} + \left[\left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r+1/2)} - \left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r)}\right] \omega \tag{A42}$$

where row $\left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r+1/2)}$ is the explicit solution to

$$\bar{\underset{\approx}{L}}_n \bar{\underset{\approx}{U}}_n \left(\bar{\underset{\sim}{P}}^{*}_n\right)^{(r+1/2)} = \left[\bar{\underset{\sim}{R}}_n - \bar{\underset{\approx}{B}}^{T}_{n-1} \left(\bar{\underset{\sim}{P}}^{*}_{n-1}\right)^{(r)}_{(r+1)} - \bar{\underset{\approx}{B}}_n \left(\bar{\underset{\sim}{P}}^{*}_{n+1}\right)^{(r)}_{(r+1)}\right] \tag{A43}$$

by forward elimination and backward substitution.

In summary, the solution of (A3) for $\underset{\sim}{P}^{*}$ by line relaxation proceeds as follows:

(1) Initialize the $\underset{\approx}{A}$ and $\underset{\approx}{B}$ matrices and the forcing vector $\underset{\sim}{R}$.

(2) Decompose each $\underset{\approx}{A}_n$ and save the $d_{k,n}$, $\bar{e}_{k,n}$ and $\bar{f}_{k,n}$ elements using Equations (A27). These elements may be stored over the $a_{k,n}$, $b_{k,n}$ and $c_{k,n}$ elements, respectively, of $\underset{\approx}{A}_n$.

(3) Transform $\underset{\sim}{R}$, $\underset{\sim}{P}^*$ and $\underset{\approx}{B}$ to $\bar{\underset{\sim}{R}}$, $\bar{\underset{\sim}{P}}^*$ and $\bar{\underset{\approx}{B}}$ using Eqs. (A36), (A37) and (A38).

(4) Apply Eqs. (A42) and (A43) iteratively until the solution is obtained. During the first-half of each pass the method solves explicitly for the $\bar{\underset{\sim}{P}}^*_n$ associated with the red rows, employing the proper forcing function and the previous pass values of $\bar{\underset{\sim}{P}}^*_n$ associated with the adjacent black rows. The individual elements of each $\bar{\underset{\sim}{P}}^*_n$ are then adjusted according to Equation (A42). During the second half pass the roles of the red and black rows are interchanged.

(5) Transform the solution $\bar{\underset{\sim}{P}}^*$ back to $\underset{\sim}{P}^*$ using Equation (A37); i.e.,

$$\underset{\sim}{P}^* = \underset{\approx}{D}^{-1} \bar{\underset{\sim}{P}}^* \quad .$$

APPENDIX B

Programming Details of Line SOR

INITIALIZE $\underset{\approx}{A}_n$ FOR $n = 1,2,3,\ldots,M$

Diagonal of $\underset{\approx}{A}_n$

$$
\begin{aligned}
a_{k,n} = A_{k,n} &+ \underline{\underline{B_{k,n}}} + \underline{\underline{B_{k+1,n}}} + \underline{\underline{C_{k,n}}} + \underline{\underline{C_{k,n-1}}} \\
&+ \underline{\underline{D_{k,n}}} + \underline{\underline{D_{k-1,n}}} + \underline{\underline{E_{k,n}}} + \underline{\underline{E_{k-2,n}}} \\
&+ \underline{\underline{F_{k,n}}} + \underline{\underline{F_{k+2,n-1}}} + \underline{\underline{G_{k,n}}} + \underline{\underline{G_{k-2,n-1}}} \\
&+ \underline{\underline{R_{k,n}}} + \underline{\underline{R_{k+2,n-1}}} + \underline{\underline{4R_{k+1,n}}} \\
&+ \underline{\underline{S_{k,n}}} + \underline{\underline{S_{k-2,n-1}}} + \underline{\underline{4S_{k-1,n}}} \\
+ \underline{\underline{16T_{k,n}}} &+ \underline{\underline{T_{k-1,n}}} + \underline{\underline{T_{k-1,n+1}}} + \underline{\underline{T_{k+1,n+1}}} + \underline{\underline{T_{k+1,n}}} \\
&+ \underline{\underline{W_{k,n}}} + \underline{\underline{W_{k,n-1}}} + \underline{\underline{W_{k-i,n}}} + \underline{\underline{W_{k+1,n}}}
\end{aligned}
$$

for k odd: $k = 1,3,5,\ldots,K$

$$
\begin{aligned}
a_{k,n} = {} & A_{k,n} + \underline{\underline{B_{k,n}}} + \underline{\underline{B_{k+1,n-1}}} + \underline{\underline{C_{k,n}}} + \underline{\underline{C_{k,n-1}}} \\
& + \underline{\underline{D_{k,n}}} + \underline{\underline{D_{k-1,n-1}}} + \underline{\underline{E_{k,n}}} + \underline{\underline{E_{k-2,n}}} \\
& + \underline{\underline{F_{k,n}}} + \underline{\underline{F_{k+2,n-1}}} + \underline{\underline{G_{k,n}}} + \underline{\underline{G_{k-2,n-1}}} \\
& + \underline{\underline{R_{k,n}}} + \underline{\underline{R_{k+2,n-1}}} + \underline{\underline{4R_{k+1,n-1}}} \\
& + \underline{\underline{S_{k,n}}} + \underline{\underline{S_{k-2,n-1}}} + \underline{\underline{4S_{k-1,n-1}}} \\
+ & \underline{\underline{16T_{k,n}}} + \underline{\underline{T_{k-1,n-1}}} + \underline{\underline{T_{k-1,n}}} + \underline{\underline{T_{k+1,n}}} + \underline{\underline{T_{k+1,n-1}}} \\
& + \underline{\underline{W_{k,n}}} + \underline{\underline{W_{k,n-1}}} + \underline{\underline{W_{k-1,n-1}}} + \underline{\underline{W_{k+1,n-1}}}
\end{aligned}
$$

for k even: $k = 2,4,6,\ldots,K-1$

First off-diagonal of $\underset{\approx}{A}_n$

$$b_{k,n} = -D_{k,n} - 2S_{k,n} - 2S_{k-1,n-1} - 4T_{k,n} - 4T_{k+1,n} - W_{k,n} - W_{k+1,n-1}$$

for k odd: $k = 1,3,\ldots,K-2$

$$b_{k,n} = -B_{k+1,n} - 2R_{k+2,n-1} - 2R_{k+1,n} - 4T_{k,n} - 4T_{k+1,n} - W_{k,n-1} - W_{k+1,n}$$

for k even: $k = 2,4,6,\ldots,K-1$

Second off-diagonal of $\underset{\approx}{A}_n$

$$c_{k,n} = -E_{k,n} + T_{k+1,n} + T_{k+1,n-1} + W_{k+1,n-1}$$

for k odd: $k = 1,3,\ldots,K-2$

$$c_{k,n} = -E_{k,n} + T_{k+1,n+1} + T_{k+1,n} + W_{k+1,n}$$

for k even: $k = 2,4,6,\ldots,K-1$

INITIALIZE $\underset{\approx}{B}_n$ FOR $n = 1,2,3,\ldots,M-1$

Diagonal of $\underset{\approx}{B}_n$

$$\alpha_{k,n} = \underline{\underline{-C_{k,n}}} + \underline{T_{k-1,n}} + \underline{T_{k+1,n}} + \underline{W_{k,n}} \qquad \text{for k odd: } k=1,3,\ldots,K$$

$$\alpha_{k,n} = \underline{\underline{-C_{k,n}}} + \underline{T_{k-1,n+1}} + \underline{T_{k+1,n+1}} + \underline{W_{k,n}} \qquad \text{for k even: } k=2,4,\ldots,K-1$$

First row above diagonal

$$\beta_{k,n} = 0 \qquad \text{for k odd: } k=1,3,\ldots,K-2$$

$$\beta_{k,n} = \underline{\underline{-D_{k,n}}} - \underline{2S_{k,n}} - \underline{2S_{k-1,n}} - \underline{4T_{k,n}}$$

$$- \underline{4T_{k+1,n+1}} - \underline{W_{k,n}} - \underline{W_{k+1,n}} \qquad \text{for k even: } k=2,4,\ldots,K-1$$

Second row above diagonal

$$\gamma_{k,n} = \underline{\underline{-G_{k,n}}} + \underline{S_{k,n}} + \underline{T_{k+1,n}} \qquad \text{for k odd: } k=1,3,\ldots,K-2$$

$$\gamma_{k,n} = \underline{\underline{-G_{k,n}}} + \underline{S_{k,n}} + \underline{T_{k+1,n+1}} \qquad \text{for k even: } k=2,4,\ldots,K-3$$

First row below diagonal

$$\delta_{k,n} = \underline{\underline{-B_{k+1,n}}} - \underline{2R_{k+1,n}} - \underline{2R_{k+2,n}} - \underline{4T_{k+1,n}} - \underline{4T_{k,n+1}}$$

$$- \underline{W_{k+1,n}} - \underline{W_{k,n}} \qquad \text{for k odd: } k=1,3,\ldots,K-2$$

$$\delta_{k,n} = 0 \qquad \text{for k even: } k=2,4,\ldots,K-1$$

Second row below diagonal

$$\epsilon_{k,n} = \underline{\underline{-\Gamma_{k+2,n}}} + \underline{R_{k+2,n}} + \underline{T_{k+1,n}} \qquad \text{for k odd: } k=1,3,\ldots,K-2$$

$$\epsilon_{k,n} = \underline{\underline{-\Gamma_{k+2,n}}} + \underline{R_{k+2,n}} + \underline{T_{k+1,n+1}} \qquad \text{for k even: } k=2,4,\ldots,K-3$$

INITIALIZE $\hat{\underset{\sim}{R}}_n$ FOR $n = 1,2,\ldots,M$

$$\hat{R}_{k,n} = A_{k,n}p_{k,n}$$

$$- \underline{\underline{B_{k,n}b_{k,n}}} + \underline{\underline{B_{k+1,n-1}b_{k+1,n-1}}} - \underline{\underline{C_{k,n}c_{k,n}}} + \underline{\underline{C_{k,n-1}c_{k,n-1}}}$$

$$- \underline{\underline{D_{k,n}d_{k,n}}} + \underline{\underline{D_{k-1,n-1}d_{k-1,n-1}}} - \underline{\underline{E_{k,n}e_{k,n}}} + \underline{\underline{E_{k-2,n}e_{k-2,n}}}$$

$$- \underline{\underline{F_{k,n}f_{k,n}}} + \underline{\underline{F_{k+2,n-1}f_{k+2,n-1}}} - \underline{\underline{G_{k,n}g_{k,n}}} + \underline{\underline{G_{k-2,n-1}g_{k-2,n-1}}}$$

$$+ \underline{R_{k,n}r_{k,n}} + \underline{R_{k+2,n-1}r_{k+2,n-1}} - \underline{2R_{k+1,n-1}r_{k+1,n-1}}$$

$$+ \underline{S_{k,n}s_{k,n}} + \underline{S_{k-2,n-1}s_{k-2,n-1}} - \underline{2S_{k-1,n-1}s_{k-1,n-1}}$$

$$- \underline{4T_{k,n}t_{k,n}} + \underline{T_{k-1,n-1}t_{k-1,n-1}} + \underline{T_{k-1,n}t_{k-1,n}} + \underline{T_{k+1,n}t_{k+1,n}} + \underline{T_{k+1,n-1}t_{k+1,n-1}}$$

$$+ \underline{W_{k,n}w_{k,n}} + \underline{W_{k,n-1}w_{k,n-1}} - \underline{W_{k-1,n-1}w_{k-1,n-1}} - \underline{W_{k+1,n-1}w_{k+1,n-1}}$$

for k odd: $k=1,3,5,\ldots,K$

$$\hat{R}_{k,n} = A_{k,n}p_{k,n}$$

$$- \underline{\underline{B_{k,n}b_{k,n}}} + \underline{\underline{B_{k+1,n}b_{k+1,n}}} \quad - \underline{\underline{C_{k,n}c_{k,n}}} + \underline{\underline{C_{k,n-1}c_{k,n-1}}}$$

$$- \underline{\underline{D_{k,n}d_{k,n}}} + \underline{\underline{D_{k-1,n}d_{k-1,n}}} \quad - \underline{\underline{E_{k,n}e_{k,n}}} + \underline{\underline{E_{k-2,n}e_{k-2,n}}}$$

$$- \underline{\underline{F_{k,n}f_{k,n}}} + \underline{\underline{F_{k+2,n-1}f_{k+2,n-1}}} - \underline{\underline{G_{k,n}g_{k,n}}} + \underline{\underline{G_{k-2,n-1}g_{k-2,n-1}}}$$

$$+ \underline{R_{k,n}r_{k,n}} + \underline{R_{k+2,n-1}r_{k+2,n-1}} - \underline{2R_{k+1,n}r_{k+1,n}}$$

$$+ \underline{S_{k,n}s_{k,n}} + \underline{S_{k-2,n-1}s_{k-2,n-1}} - \underline{2S_{k-1,n}s_{k-1,n}}$$

$$- \underline{4T_{k,n}t_{k,n}} + \underline{T_{k-1,n}t_{k-1,n}} + \underline{T_{k-1,n+1}t_{k-1,n+1}} + \underline{T_{k+1,n+1}t_{k+1,n+1}} + \underline{T_{k+1,n}t_{k+1,n}}$$

$$+ \underline{W_{k,n}w_{k,n}} + \underline{W_{k,n-1}w_{k,n-1}} - \underline{W_{k-1,n}w_{k-1,n}} - \underline{W_{k+1,n}w_{k+1,n}}$$

for k even: $k = 2,4,6,\ldots,K-1$

COMPUTE $\underset{\approx}{D}_n$ AND OFF-DIAGONALS OF $\underset{\approx}{\bar{L}}_n$ FOR $n = 1,2,\ldots,M$

Need to save only $d_{k,n}$, $\bar{e}_{k,n}$, $\bar{f}_{k,n}$ (write over $a_{k,n}$, $b_{k,n}$, $c_{k,n}$)

Initialize:

$$d_{1,n} = \sqrt{a_{1,n}} \quad \Rightarrow \text{ save}$$

$$e_{1,n} = \frac{b_{1,n}}{d_{1,n}}$$

$$d_{2,n} = \sqrt{a_{2,n} - e_{1,n}^2} \quad \Rightarrow \text{ save}$$

$$\bar{e}_{1,n} = \frac{e_{1,n}}{d_{2,n}} \quad \Rightarrow \text{ save}$$

Iterate: $k = 1,2,\ldots,K-2$

$$f_{k,n} = \frac{c_{k,n}}{d_{k,n}}$$

$$e_{k+1,n} = \frac{b_{k+1,n} - e_{k,n} f_{k,n}}{d_{k+1,n}}$$

$$d_{k+2,n} = \sqrt{a_{k+2,n} - e_{k+1,n}^2 - f_{k,n}^2} \quad \Rightarrow \text{ save}$$

$$\bar{e}_{k+1,n} = \frac{e_{k+1,n}}{d_{k+2,n}} \quad \Rightarrow \text{ save}$$

$$\bar{f}_{k,n} = \frac{f_{k,n}}{d_{k+2,n}} \quad \Rightarrow \text{ save}$$

TRANSFORM $\underset{\sim}{\bar{R}}_n = \underset{\approx}{D}_n^{-1} \underset{\sim}{\hat{R}}_n$ FOR $n = 1,2,\ldots,M$

$$\bar{R}_{k,n} = \frac{\hat{R}_{k,n}}{d_{k,n}} \quad ; \quad k = 1,2,\ldots,K$$

COMPUTE $\underset{\approx}{\overline{B}}_n$ FOR $n = 1,2,\ldots,M-1$

$$\overline{\alpha}_{k,n} = \frac{\alpha_{k,n}}{d_{k,n}\, d_{k,n+1}} \qquad \text{for } k = 1,2,\ldots,K$$

$$\overline{\beta}_{k,n} = \frac{\beta_{k,n}}{d_{k,n}\, d_{k+1,n+1}} \qquad \text{for } k = 1,2,\ldots,K-1$$

$$\overline{\gamma}_{k,n} = \frac{\gamma_{k,n}}{d_{k,n}\, d_{k+2,n+1}} \qquad \text{for } k = 1,2,\ldots,K-2$$

$$\overline{\delta}_{k,n} = \frac{\delta_{k,n}}{d_{k+1,n}\, d_{k,n+1}} \qquad \text{for } k = 1,2,\ldots,K-1$$

$$\overline{\epsilon}_{k,n} = \frac{\epsilon_{k,n}}{d_{k+2,n}\, d_{k,n+1}} \qquad \text{for } k = 1,2,\ldots,K-2$$

TRANSFORM FIRST GUESS FOR ODD LINES: $n = 1,3,\ldots,M$

$$(\bar{p}_o^*)_{k,n} = d_{k,n}\ (p_o^*)_{k,n} \qquad \text{for } k = 1,2,\ldots,K$$

Note: First guess for even lines computed using line relaxation (i.e., the following steps with n even) with $\omega \equiv 1$.

ITERATE THE FOLLOWING STEPS UNTIL CONVERGED

First half pass: $n = 1,3,5,\ldots,K$

Second half pass: $n = 2,4,6,\ldots,K-1$

Compute $\underset{\sim}{F}_n$

$$F_{k,n} = \bar{R}_{k,n} = \begin{vmatrix} \bar{\gamma}_{k-2,n-1} & \bar{p}^*_{k-2,n-1} \\ \bar{\beta}_{k-1,n-1} & \bar{p}^*_{k-1,n-1} \\ \bar{\alpha}_{k,n-1} & \bar{p}^*_{k,n-1} \\ \bar{\delta}_{k,n-1} & \bar{p}^*_{k+1,n-1} \\ \bar{\epsilon}_{k,n-1} & \bar{p}^*_{k+2,n-1} \end{vmatrix} - \begin{vmatrix} \bar{\epsilon}_{k-2,n} & \bar{p}^*_{k-2,n+1} \\ \bar{\delta}_{k-1,n} & \bar{p}^*_{k-1,n+1} \\ \bar{\alpha}_{k,n} & \bar{p}^*_{k,n+1} \\ \bar{\beta}_{k,n} & \bar{p}^*_{k+1,n+1} \\ \bar{\gamma}_{k,n} & \bar{p}^*_{k+2,n+1} \end{vmatrix}$$

for $k = 1,2,3,\ldots,K$

Note: Terms referencing indices outside of boundaries ($n < 1$, $n > M$, $k < 1$, $k > K$) are set to zero.

Solve $\underset{\approx}{\bar{L}}_n \; \underset{\approx}{\bar{U}}_n \; \underset{\sim}{\bar{p}}^*_n = \underset{\sim}{\Gamma}_n$ for $\underset{\sim}{\bar{p}}^*_n$

Step 1 Solve $\underset{\approx}{\bar{L}}_n \; \underset{\sim}{Z}_n = \underset{\sim}{\Gamma}_n$ for $\underset{\sim}{Z}_n$

Initialize:

$$Z_{1,n} = \Gamma_{1,n}$$

$$Z_{2,n} = \Gamma_{2,n} - \bar{e}_{1,n} \; Z_{1,n}$$

Iterate: $k = 3,4,\ldots,K$

$$Z_{k,n} = \Gamma_{k,n} - \bar{e}_{k-1,n} \; Z_{k-1,n} - \bar{f}_{k-2,n} \; Z_{k-2,n}$$

Step 2 Solve $\underset{\approx}{U}_n \; \underset{\sim}{\bar{p}}^*_n = \underset{\sim}{Z}_n$ for $\underset{\sim}{\bar{p}}^*_n$

Initialize:

$$\bar{p}^*_{K,n} = Z_{K,n}$$

$$\bar{p}^*_{K-1,n} = Z_{K-1,n} - \bar{e}_{K-1,n} \; \bar{p}^*_{K,n}$$

Iterate: $k = K-2, K-3, \ldots, 2, 1$

$$\bar{p}^*_{k,n} = Z_{k,n} - \bar{e}_{k,n} \; \bar{p}^*_{k+1,n} - \bar{f}_{k,n} \; \bar{p}^*_{k+2,n}$$

This becomes $\left(\underset{\sim}{\bar{p}}^*_n\right)^{(r+1/2)}$

Correct a line of $\bar{p}^*_{k,n}$; i.e., correct $\left(\underset{\sim}{\bar{p}}^*_n\right)^r$

$$\left(\underset{\sim}{\bar{p}}^*_n\right)^{(r+1)} = \left(\underset{\sim}{\bar{p}}^*_n\right)^r + \left[\left(\underset{\sim}{\bar{p}}^*_n\right)^{(r+1/2)} - \left(\underset{\sim}{\bar{p}}^*_n\right)^r\right] \omega$$

WHEN CONVERGED

$$p^*_{k,n} = \bar{p}^*_{k,n} / d_{k,n}$$

for $k = 1,2,3,\ldots,K$
and $n = 1,2,3,\ldots,M$

Boundary Conditions on Spreading Parameter Weights in (k,n) Grid Notation

For the following (k,n) the indicated weights are set = 0.

I. First-Difference Weights

	k	n	
B:	1	1,2,3,...,M	first column on left
	1,2,3,...,K	M	top row
C:	2,4,6,...,K-1	M-1	row below top row
	1,2,3,...,K	M	top row
D:	K	1,2,3,...,M	first column on right
	1,2,3,...,K	M	top row
E:	K-1,K	1,2,3,...,M	rightmost two columns
	2,4,6,...,K-1	M	top row
F:	1,2	1,2,3,...,M	leftmost two columns
	2,4,6,...,K-1	M-1	row below top row
	1,2,3,...,K	M	top row
G:	K-1,K	1,2,3,...,M	rightmost two columns
	2,4,6,...,K-1	M-1	row below top row
	1,2,3,...,K	M	top row

II. Higher-Order Spreading Parameter Weights

R:	same as F		
S:	same as G		
T:	1,3,5,...,K	1	bottom row
	1,2,3,...,K	M	top row
	1,K	1,2,3,...,M	left and right columns
W:	1,K	1,2,3,...,M	left and right columns
	1,2,3,...,K	M	top row
	2,4,6,...,K-1	M-1	row below top row

Spreading Parameter Estimates in (k,n) Notation

Parameter	Estimate for corner point (k odd, m integer)	Estimate for center point (k even, m half-integer)
First Differences:		
$b_{k,n}$	$p_{k-1,n} - p_{k,n}$	$p_{k-1,n+1} - p_{k,n}$
$c_{k,n}$	$p_{k,n+1} - p_{k,n}$	same
$d_{k,n}$	$p_{k+1,n} - p_{k,n}$	$p_{k+1,n+1} - p_{k,n}$
$e_{k,n}$	$p_{k+2,n} - p_{k,n}$	same
$f_{k,n}$	$p_{k-2,n+1} - p_{k,n}$	same
$g_{k,n}$	$p_{k+2,n+1} - p_{k,n}$	same
Second Differences:		
$r_{k,n}$	$p_{k-2,n+1} + p_{k,n} - 2p_{k-1,n}$	$p_{k-2,n+1} + p_{k,n} - 2p_{k-1,n+1}$
$s_{k,n}$	$p_{k+2,n+1} + p_{k,n} - 2p_{k+1,n}$	$p_{k+2,n+1} + p_{k,n} - 2p_{k+1,n+1}$
Laplacian:		
$t_{k,n}$	$p_{k-1,n} + p_{k+1,n} + p_{k-1,n-1} + p_{k+1,n-1} - 4p_{k,n}$	$p_{k-1,n} + p_{k+1,n} + p_{k-1,n+1} + p_{k+1,n+1} - 4p_{k,n}$
Cross Difference:		
$w_{k,n}$	$p_{k,n+1} + p_{k,n} - p_{k-1,n} - p_{k+1,n}$	$p_{k,n+1} + p_{k,n} - p_{k-1,n+1} - p_{k+1,n+1}$

Assembly Module Coordinates (k,n) of $p_{i,j}$ and $\underset{\sim}{V}_{i,j}$ Reports

Scalar	(k,n)
p	$k = \left[\left(j+i\right)_R - \left(j-i\right)_R\right] - 1$
	$n = \left\{\frac{1}{2}\left[\left(j+i\right)_R + \left(j-i\right)_R\right]\right\}_T$

First-Difference Parameter	(k,n)
b	$k = \left[\left(j+i\right)_R - \left(j-i\right)_T\right] - 1$
	$n = \left\{\frac{1}{2}\left[\left(j+i\right)_R + \left(j-i\right)_T\right]\right\}_T$
d	$k = \left[\left(j+i\right)_T - \left(j-i\right)_R\right] - 1$
	$n = \left\{\frac{1}{2}\left[\left(j+i\right)_T + \left(j-i\right)_R\right]\right\}_T$
f	$k = \left(j+i\right)_R - \left(j-i\right)_R$
	$n = \left\{\frac{1}{2}\left[\left(j+i\right)_R + \left(j-i\right)_R - 1\right]\right\}_T$
g	$k = \left[\left(j+i\right)_R - \left(j-i\right)_R - 2\right]$
	$n = \left\{\frac{1}{2}\left[\left(j+i\right)_R + \left(j-i\right)_R - 1\right]\right\}_T$
c	$k = \left(2i\right)_R - 1$
	$n = \left\{j - \frac{1}{2}\,\mathrm{MOD}\left[\left(2i\right)_R, 2\right]\right\}_T$
e	$k = \left(2i_T - 1\right) + \mathrm{ISIGN}\left[\mathrm{MOD}\left\{\left(2j\right)_R, 2\right\}, \left(i - i_T - \frac{1}{2}\right)\right]$
	$n = \left[j + 0.25\right]_T$

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